

## Level Curves

Suppose  $T(x,y) = 1 - x^2 - y^2$  gives the temperature at any pt  $(x,y)$  in a region of the  $x$ - $y$  plane.

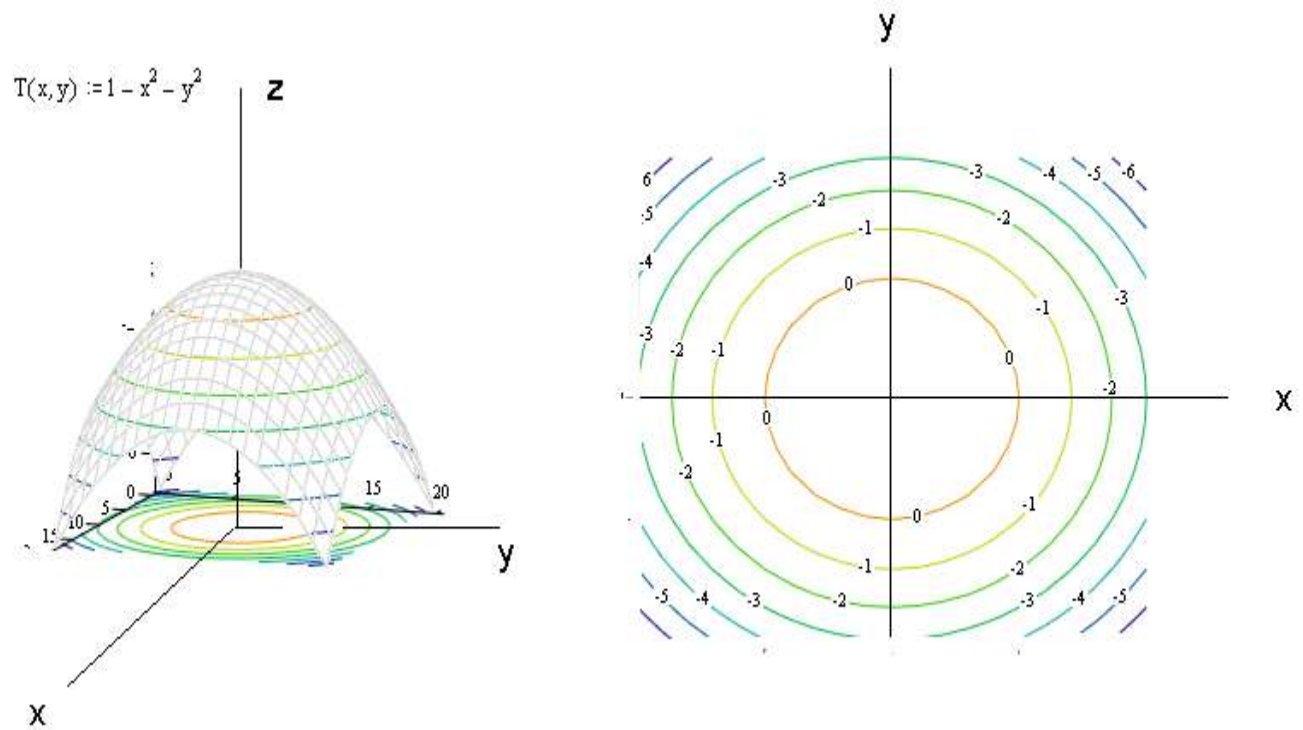
Then a level curve or contour of  $T$  is a curve in the  $xy$  plane such that at every pt on that curve  $T$  has the same temperature.

This is to say a level curve is the intersection of the surface  $z = f(x,y)$  and the horizontal plane  $z = k$  projected into the  $xy$  plane.

A contour diagram is a set of level curves for values of  $z$  in equal increments.

Note for the function  $T(x,y) = 1 - x^2 - y^2$  the contours  $z = k$  are the circles  $x^2 + y^2 = 1 + k$ .

See the diagrams below:

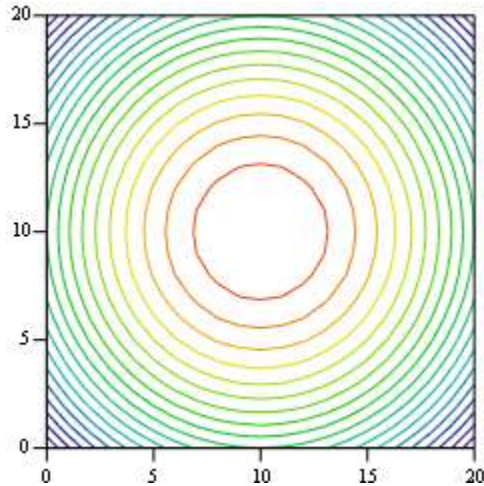


Creating Contour Diagrams in Mathcad is very simple as it uses exactly the same template as for creating 3-Dgraphs. Double click on the graph to bring up the FORMAT window. Using the

GENERAL TAB change the DISPLAY AS to Contour Plot.

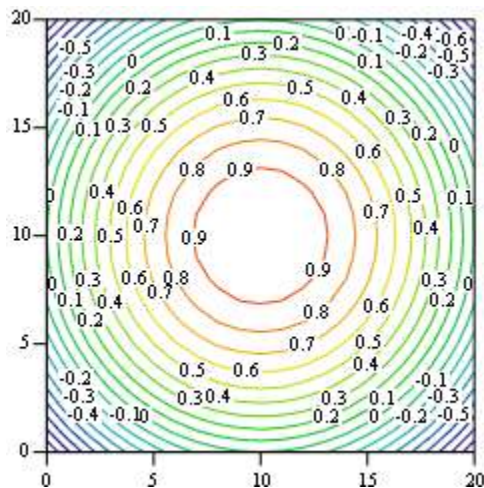
Let's see how to create the diagrams above.

```
a := -1  b := 1  c := -1  d := 1  Δx := .1  Δy := .1
i := 0..  $\frac{b-a}{\Delta x}$   j := 0..  $\frac{d-c}{\Delta y}$   xi := a + i·Δx  yj := c + j·Δy
f(x,y) := 1 - x2 - y2  Mi,j := f(xi, yj)
```



M

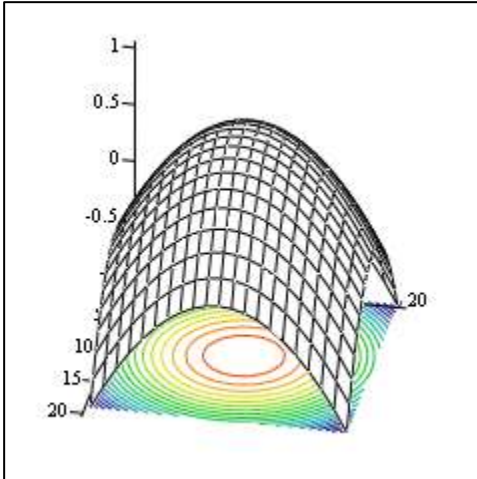
Now we want to number the Contours. Again go to the FORMAT WINDOW . Use the SPECIAL TAB and click on numbered under Contour Options. You can then either use AUTO CONTOUR or click this off and put in as many contours as you want,



M

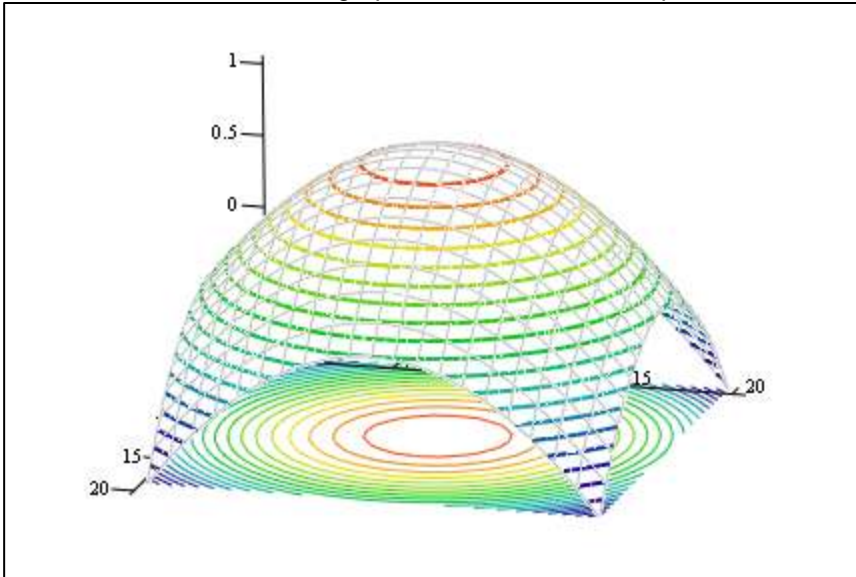
How do we get the surface and the Contour Diagram on the same Graph? Simply Graph the way we have been doing but put another M in the place Holder on the Graph. Use the GENERAL tab and change one to a surface plot and the other to a contour plot.

Here Plot 1 is displayed as a Surface Plot (solid color hide lines turned on) and Plot2 is displayed as a contour plot (color plot) Note when including a contour plot on a surface you can't use numbering.



M, M

So How did I create the first graph in this Lab? Use 3 copies of M



M, M, M

Plot 1

Surface Plot - Appearance :Solid Color (light gray), Hide lines turned on

Plot 2

Contour Plot - Appearance Color Map

Plot 3

Surface Plot-- Appearance: Color Map, Line Option changed to Contour Lines, Weight changed to 2( to make them look thicker), Hide Lines turned on

### Exercise 1

1. Make a contour plot for  $f(x, y) := 2 - x^2 - y^2$

Format as follows :

- Graph on the rectangle  $[-10, 10] \times [-10, 10]$  with a step size of 1 in each direction.
- Auto Contour off
- No. of contours 10

2. Make a plot with the surface and contour diagram on the same plot

### Exercise 2

1. Make a contour plot for  $f(x, y) := |x| |y|$

Format as follows :

- Graph on the square  $[-1, 1] \times [-1, 1]$  with a step size of .05 in each direction.
- Auto Contour off -Try various numbers of contours

2. Make a plot with the surface and contour diagram on the same plot

### Exercise 3

Make a contour plot for  $f(x, y) := \frac{x^2}{4} + \frac{y^2}{25}$

Format as follows :

- Graph on the square  $[-10, 10] \times [-10, 10]$  with a step size of .5 in each direction.
- Make a plot with the surface and contour diagram on the same plot

### Exercise 4

Make a contour plot for  $f(x, y) := \sin(x + y)$

Graph on the square  $[-1, 1] \times [-1, 1]$  --You decide the formatting

Notice your contour map consists of a rectangular strip through the center, centered on the diagonal of the square  $[-1, 1] \times [-1, 1]$ . How do you account for this?

### Exercise 5

Make a contour plot for  $f(x, y) := \frac{1}{1 + \sqrt{|y - \sin(x)|} + \sqrt{|x + \sin(y)|}}$  on the square  $[-3, 3] \times [-3, 3]$

Use a step size of 0.1. Because the function increases so rapidly turn numbering off and auto contour off but change the number of contours to 30.

Graph the contour diagram and the Surface separately. Graph one copy and Paste it next to the other and make changes.

See what happens if you click on Fill under Contour Options.