Animating Vector Value Functions in 3 Space

In this discussion we want to develop the template for generating animations of vector valued functions in 3 space. You may want to consider the animations on the vector value function page on Calculus7.com to remind yourself what we are leading up to. At the end I will also describe how to add in the x,y, and z axes.

Recall that the basic idea of a vector valued function is that instead of thinking of a curve in 3 space as being generated by 3 parametric equations x(t),y(t), and z(t) it is generated by a single vector equation:

$\overrightarrow{r(t)} = x(t)\cdot\vec{i} + y(t)\cdot\vec{j} + z(t)\cdot\vec{k}$ where $\overrightarrow{r(t)}$ is a vector from the origin to the point (x(t),y(t),z(t)) for each t.

The curve is then swept out by the terminal points of $\overrightarrow{r(t)}$ as t increases .

So we need to simply develop a way of graphing r(t) for each t.

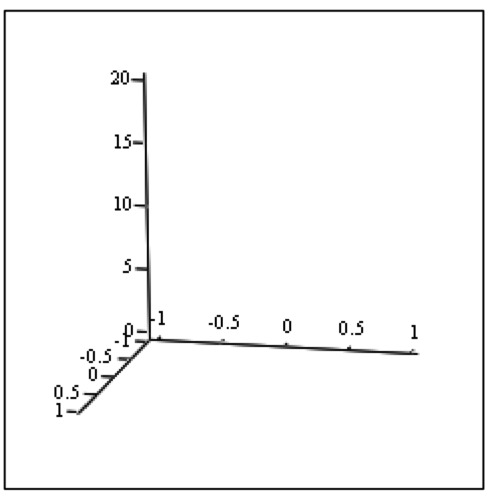
We saw that to animate parametric equations in 3- space the template is :

$s := 0..FRAME$

$t(s) := s\cdot.1$

$i := 0..1$

$x_{i,s} := \cos(t(s))$ $y_{i,s} := \sin(t(s))$ $z_{i,s} := t(s)$



(x, y, z)

We also saw that to animate 2 sets of parametric equations the template is:

$s := 0..FRAME$

$t(s) := s \cdot .1$

$i := 0..1$

$x_{i,s} := \cos(t(s))$ $y_{i,s} := \sin(t(s))$ $z_{i,s} := t(s)$

$x1_{i,s} := \cos(t(s))$ $y1_{i,s} := -\sin(t(s))$ $z1_{i,s} := t(s)$

$(x, y, z), (x1, y1, z1)$

The setup starts the same with one very significant difference. We still want $s := 0..FRAME$ to graph the curve. We also define $s1 := FRAME$ so we can generate the line segment from the origin to the point (x(t(s)), y(t(s)),z(t(s)) (Of course you can use any variable not necessarily s1)

We then define: (for the helix but obviously you can use any 3 functions you want depending on the curve)

$xv(t(s1) := \cos(t(s1))$ $yv(t(s1) := \sin(t(s1))$ $zv(t(s1) := t(s1)$ I use xv etc to indicate this is part of a vector but again you choose just don't use x,y, and z again.

The complete set up is

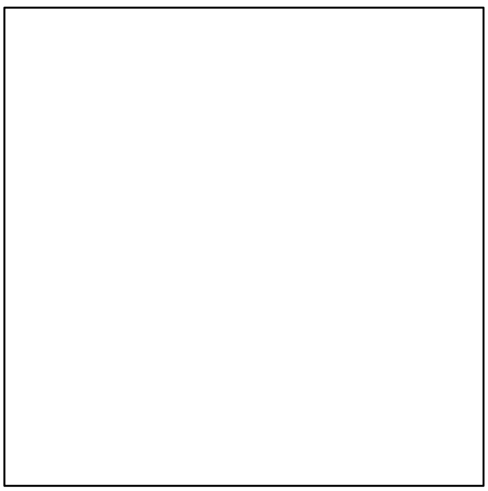
$s := 0..FRAME$ $\quad s1 := FRAME$

$t(s) := s \cdot .1$

$i := 0..1$

$x_{i,s} := \cos(t(s))$ $\quad y_{i,s} := \sin(t(s))$ $\quad z_{i,s} := t(s)$

$xv_{i,s1} := \cos(t(s1))$ $\quad yv_{i,s1} := \sin(t(s1))$ $\quad zv_{i,s1} := t(s1)$



$(x, y, z), (xv, yv, zv)$

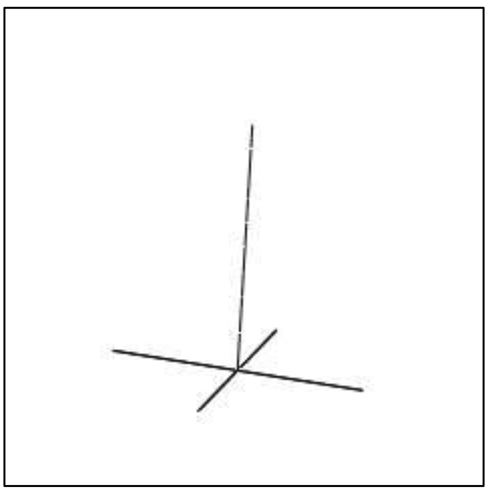
See Animation Vector Valued 1

Let's refine our work by adding in the x, y and z axes.
This is making sausage and isn't pretty but adds a lot to the animation.
In our helix example x and y go from -1 to 1 and z goes from 0 to 20. We Define X1,Y1, and Z1 as below

$i := 0..1 \quad m := 0..4($

$X1_{i,m} := 0 \quad Y1_{i,m} := (\quad Z1_{i,m} := (m - 20)$

Note Z varies from -20 to 20 but since we fixed the z axis from 0 to 20 only that part will appear.
So we don't have to define 9 functions we permute the variables as below to graph the axes.
The order is z axis, y-axis, x-axis. Note since we fixed the x and y axes the axes on the graph will only appear for the ranges we want



$(X1, Y1, Z1), (Y1, Z1, X1), (Z1, X1, Y1)$

So now putting it all together:

$s := 0..FRAMI \quad s1 := FRAMI$

$t(s) := s \cdot .1 \quad i := 0..1 \quad m := 0..4($

$x_{i,s} := \cos(t(s)) \quad y_{i,s} := \sin(t(s)) \quad z_{i,s} := t(s)$

$xv_{i,s1} := \cos(t(s1)) \quad yv_{i,s1} := \sin(t(s1)) \quad zv_{i,s1} := t(s1)$

$X1_{i,m} := 0 \quad Y1_{i,m} := (\quad Z1_{i,m} := (m - 20)$

$(x, y, z), (xv, yv, zv), (X1, Y1, Z1), (Y1, Z1, X1), (Z1, X1, Y1)$

See Animation Vector Valued 2