Creating a Vector Valued Function with the Unit Tangent and Unit Normal

Once again we'll consider a helix

Position Vector

$s1 := FRAMI$

$t(s1) := s1 \cdot .1$

$i := 0..1$

Curve

$s := 0..FRAMI$

$t1(s) := s \cdot .1$

$x1_{i,s1} := \cos(t1(s1))$ $y1_{i,s1} := \sin(t1(s1))$ $z1_{i,s1} := t1(s1)$ $x_{i,s} := \cos(t(s))$ $y_{i,s} := \sin(t(s))$ $z_{i,s} := t(s)$

Axes

$m := 0..4($ $X1_{i,m} := 0$ $Y1_{i,m} := ($ $Z1_{i,m} := (m - 10)$

What's new are the Tangent and Normal Vectors which we want to have as initial point

$(\cos(t1(s1)), \sin(t1(s1)), t1(s1))$ at each $t(s1)$ instead of the origin.

Recall that for a line that goes through (x_0, y_0, z_0) parallel to the vector $a \cdot \vec{i} + b \cdot \vec{j} + c \cdot \vec{k}$

The equations are $x = x_0 + a \cdot t$

$y = y_0 + b \cdot t$

$z = z_0 + c \cdot t$

For the tangent vector it is obviously parallel to $\dfrac{\vec{dr}}{dt}$.

The normal vector is parallel to $\overrightarrow{\dfrac{d^2 r}{dt^2}}$

Therefore we have (using the Greek tau τ since t has already been used)

Tangent vector

$\tau := 0..1$

$xT_{i,\tau} := \cos(t(s1)) - \sin(t(s1))\cdot\tau \quad yT_{i,\tau} := \sin(t(s1)) + \cos(t(s1))\cdot\tau \quad zT_{i,\tau} := t(s1) + \tau$

Normal Vector

$xN_{i,\tau} := \cos(t(s1)) - \cos(t(s1))\cdot\tau \quad yN_{i,\tau} := \sin(t(s1)) - \sin(t(s1))\cdot\tau \quad zN_{i,\tau} := t(s1)$

Putting it all together:

$s1 := \text{FRAMI} \qquad s := 0..\text{FRAMI}$

$t(s1) := s1\cdot.1 \qquad t1(s) := s\cdot.1$

$i := 0..1$

$x1_{i,s1} := \cos(t1(s1)) \quad y1_{i,s1} := \sin(t1(s1)) \quad z1_{i,s1} := t1(s1) \qquad x_{i,s} := \cos(t(s)) \quad y_{i,s} := \sin(t(s)) \quad z_{i,s} := t(s)$

$m := 0..4(\qquad X1_{i,m} := 0 \qquad Y1_{i,m} := (\qquad Z1_{i,m} := (m - 10)$

$\tau := 0..1$

$xT_{i,\tau} := \cos(t(s1)) - \sin(t(s1))\cdot\tau \quad yT_{i,\tau} := \sin(t(s1)) + \cos(t(s1))\cdot\tau \quad zT_{i,\tau} := t(s1) + \tau$

$xN_{i,\tau} := \cos(t(s1)) - \cos(t(s1))\cdot\tau \quad yN_{i,\tau} := \sin(t(s1)) - \sin(t(s1))\cdot\tau \quad zN_{i,\tau} := t(s1)$

$(x, y, z), (x1, y1, z1), (X1, Y1, Z1), (Y1, Z1, X1), (Z1, X1, Y1), (xT, yT, zT), (xN, yN, zN)$

1.To animate I would use 200 Frames at 10 frames/sec

2. For the position, tangent and normal vectors change to red.

3. If you want the helix on the graph and just want to see the position, velocity, and acceleration vectors animated change $s := 0..FRAME$ to $s := 0..200$.