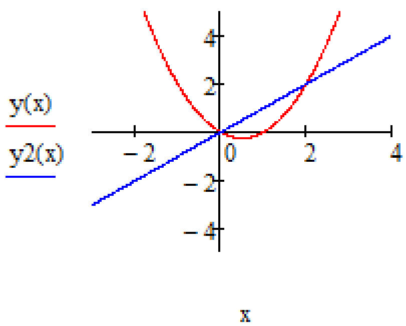
Minimization of Area

Let $y1(x) = a^3x^2 - a^4x$ and $y2(x) = x$
a. Find the value of a minimizes the area
b. What is the minimum area?

$y(x) := a^3 \cdot x^2 - a^4 \cdot x$ $\quad y2(x) := x$ below is the graph for a = 1 (not necessarily the minimum)



The integration limits are the points of intersection

$a^3 \cdot x^2 - a^4 \cdot x = x$ which yields $\dfrac{a^4 + 1}{a^3}$ and 0

$$A = \int_0^{\frac{a^4+1}{a^3}} \left[x - \left(a^3 \cdot x^2 - a^4 \cdot x\right)\right] dx = \int_0^{\frac{a^4+1}{a^3}} \left[x \cdot \left(1 + a^4\right) - a^3 \cdot x^2\right] dx = \left[\frac{x^2}{2}\left(1 + a^4\right) - \frac{a^3}{3} \cdot x^3\right] \cdot \Bigg|_0^{\frac{a^4 + 1}{a^3}}$$

$$\left[\frac{x^2}{2}\left(1 + a^4\right) - \frac{a^3}{3} \cdot x^3\right] \cdot \Bigg|_0^{\frac{a^4 + 1}{a^3}} = \frac{\left(a^4 + 1\right)^3}{2 \cdot a^6} - \frac{\left(a^4 + 1\right)^3}{3 \cdot a^6} = \frac{\left(a^4 + 1\right)^3}{6 \cdot a^6}$$

$$A = \frac{\left(a^4 + 1\right)^3}{6 \cdot a^6}$$

To minimize we set $\frac{dA}{da} = 0$

$$\frac{dA}{da} = \frac{2\cdot\left(a^4+1\right)^2}{a^3} - \frac{\left(a^4+1\right)^3}{a^7}$$

$$\frac{2\cdot\left(a^4+1\right)^2}{a^3} = \frac{\left(a^4+1\right)^3}{a^7} = 0$$

$$\frac{2}{a^3} = \frac{a^4+1}{a^7}$$

$$2\cdot a^4 = a^4 + 1$$

$$a = 1$$

$$\int_0^2 \left[x - \left(1\cdot x^2 - 1\cdot x\right)\right] dx \rightarrow \frac{4}{3}$$