Triple Integrals in Spherical Coordinates

All the Animations work basically the same so we'll consider the situation where we want the volume inside the cone $\phi = \frac{\pi}{4}$ and the sphere ρ = 1. Then we'll discuss how to animate other situations.

We need to allow ϕ vary from 0 to $\frac{\pi}{4}$ keeping θ fixed then allow θ to vary from 0 to 2π.

$$j := \begin{cases} (0..FRAME) & \text{if } FRAME \le 6 \\ (0..6) & \text{otherwise} \end{cases}$$

$$\phi_j := \pi \cdot \frac{j}{24}$$

$$i := \begin{cases} 0 & \text{if } FRAME \le 6 \\ (0..FRAME - 6) & \text{otherwise} \end{cases}$$

$$\theta_i := \frac{\pi}{24} \cdot j$$

Now is where we define $\rho_{i,j} := 1$.

We'll put in the axes

$m := 0..20$

$X1_{i,m} := 0 \qquad Y1_{i,m} := 0 \qquad Z1_{i,m} := (m - 10)$

We define the sphere

$$X_{i,j} := \rho_{i,j} \cdot \cos(\theta_i) \cdot \sin(\phi_j) \qquad Y_{i,j} := \rho_{i,j} \cdot \sin(\theta_i) \cdot \sin(\phi_j) \qquad Z_{i,j} := \rho_{i,j} \cdot \cos(\phi_j)$$

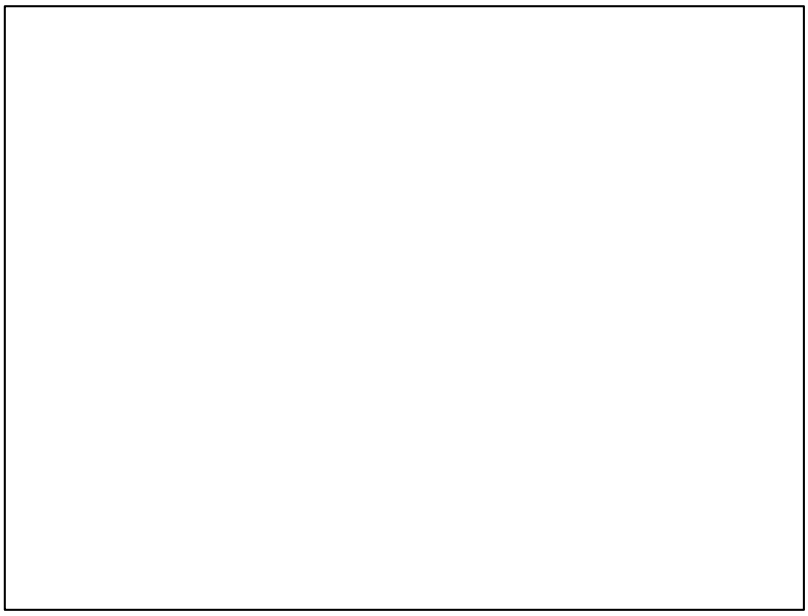
We'll define the cone-- by graphing the line $\phi = \frac{\pi}{4}$. The cone will then be generated by letting θ vary from 0 to 2π .

$s := 0..10 \quad t(s) := s \cdot .1$

$$x_{i,s} := \cos(\theta_i) \cdot \sin\left(\frac{\pi}{4}\right) \cdot t(s) \qquad y_{i,s} := \sin(\theta_i) \cdot \sin\left(\frac{\pi}{4}\right) \cdot t(s) \qquad z_{i,s} := \cos\left(\frac{\pi}{4}\right) \cdot t(s)$$

Under Format and Appearance make plots 1 and 5 color plots. Make Plots 2,3, and 4 black.

Set the x and y axes to go from -1 to 1 and the z axis to go from 0 to 1To animate use 54 frames.



$(X, Y, Z), (X1, Y1, Z1), (Z1, X1, Y1), (X1, Z1, Y1), (x, y, z)$

Using this same format we can generate several other examples simply by adjusting i and j

For example we want the volume of the upper half of a sphere We want j to vary from 0 to 12 and then use 72 Frames

Eliminate (x,y,z) from the graph as we no longer want the cone. Under the General tab under Axes Style click on equal scales

$$j := \begin{cases} (0..FRAME) & \text{if } FRAME \le 12 \\ (0..12) & \text{otherwise} \end{cases}$$

$$\phi_j := \pi \cdot \frac{j}{24}$$

$$i := \begin{cases} 0 & \text{if } FRAME \le 12 \\ (0..FRAME - 12) & \text{otherwise} \end{cases}$$

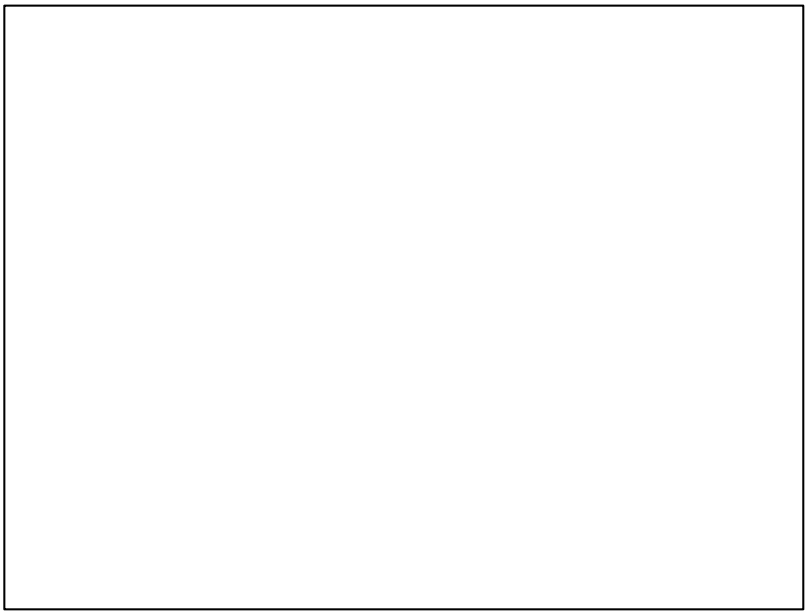
$$\theta_i := \frac{\pi}{24} \cdot i$$

$\rho_{i,j} := 1$.

$m := 0..2$

$X1_{i,m} := 0$ $Y1_{i,m} := 0$ $Z1_{i,m} := (m - 1)$

$X_{i,j} := \rho_{i,j} \cdot \cos(\theta_i) \cdot \sin(\phi_j)$ $Y_{i,j} := \rho_{i,j} \cdot \sin(\theta_i) \cdot \sin(\phi_j)$ $Z_{i,j} := \rho_{i,j} \cdot \cos(\phi_j)$



$(X, Y, Z), (X1, Y1, Z1), (Z1, X1, Y1), (X1, Z1, Y1)$

If we want that portion of the sphere in the first octant use 12 frames and :

$$j := \begin{cases} (0..FRAME) & \text{if } FRAME \le 6 \\ (0..6) & \text{otherwise} \end{cases}$$

$$\phi_j := \pi \cdot \frac{j}{24}$$

$$i := \begin{cases} 0 & \text{if } FRAME \le 6 \\ (0..FRAME - 6) & \text{otherwise} \end{cases}$$

$$\theta_i := \frac{\pi}{24} \cdot i$$