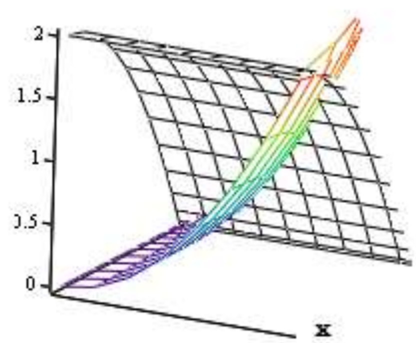
Find the volume of the solid bounded by the parabaloid $z = 4 \cdot x^2 + y^2$

and the parabolic cylinder $z = 2 - y^2$

Even though we don't really need the diagrams I've included them to help understand a little better.

The graphs are only in the first octant since by symmetry we can compute this volume and multiply by 4.

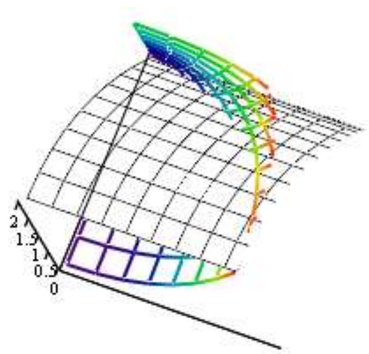
In the first Diagram we see the limits on z we go from the parabaloid to the cylinder



$$\int_{\blacksquare}^{\blacksquare}\int_{\blacksquare}^{\blacksquare}\int_{4\cdot x^2+y^2}^{2-y^2} 1\,dz\,d\blacksquare\,d\blacksquare$$

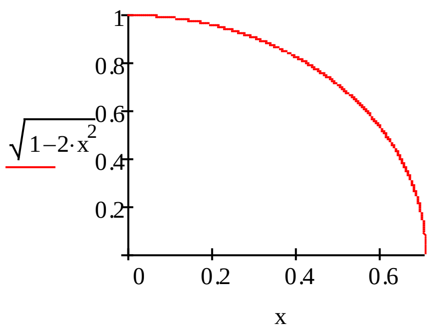
The next is a top view which tells us how to go about obtaining the curve in the x-y plane

which will gives the outer integration limits. We simply set the 2 equations for z equal to each other:



We set $2 - y^2 = 4 \cdot x^2 + y^2$

$$1 = 2 \cdot x^2 + y^2$$



We get then:

$$4 \cdot \int_0^{\frac{1}{\sqrt{2}}} \int_0^{\sqrt{1-2\cdot x^2}} \int_{4\cdot x^2+y^2}^{2-y^2} 1\,dz\,dy\,dx$$

The integrals aren't quite as bad as they look

$$4 \cdot \int_0^{\frac{1}{\sqrt{2}}} \int_0^{\sqrt{1-2\cdot x^2}} \int_{4\cdot x^2+y^2}^{2-y^2} 1\,dz\,dy\,dx = 4 \cdot \int_0^{\frac{1}{\sqrt{2}}} \int_0^{\sqrt{1-2\cdot x^2}} 2 - 4 \cdot x^2 - 2 \cdot y^2\,dy\,dx$$

$$4 \cdot \int_0^{\frac{1}{\sqrt{2}}} \int_0^{\sqrt{1-2\cdot x^2}} 2 - 4 \cdot x^2 - 2 \cdot y^2\,dy\,dx = 4 \cdot \int_0^{\frac{1}{\sqrt{2}}} \left(2 - 4 \cdot x^2\right) \cdot \sqrt{1 - 2 \cdot x^2} - \frac{2}{3} \cdot \left(1 - 2 \cdot x^2\right)^{\frac{3}{2}}\,dx$$

$$4 \cdot \int_0^{\frac{1}{\sqrt{2}}} \left(2 - 4 \cdot x^2\right) \cdot \sqrt{1 - 2 \cdot x^2} - \frac{1}{3} \cdot \left(1 - 2 \cdot x^2\right)^{\frac{3}{2}} \, dx = 4 \cdot \int_0^{\frac{1}{\sqrt{2}}} \frac{4}{3} \cdot \left(1 - 2 \cdot x^2\right)^{\frac{3}{2}} \, dx$$

$$4 \cdot \int_0^{\frac{\sqrt{2}}{2}} \frac{4 \cdot \left(1 - 2 \cdot x^2\right)^{\frac{3}{2}}}{3} \, dx$$

Use the trig substitution $x = \frac{1}{\sqrt{2}} \cdot \sin(\theta)$

We obtain:

$$4 \cdot \frac{4}{3 \cdot \sqrt{2}} \cdot \int_0^{\frac{\pi}{2}} \cos(\theta)^4 \, d\theta \rightarrow \frac{\pi \cdot \sqrt{2}}{2}$$