Triple Integrals in Cylindrical Coordinates

Here we'll develop the animation for computing the volume of the paraboloid

$$z = 1 - x^2 - y^2$$

over the unit circle r = 1. We need to keep θ fixed while we generate the cross section

for y = 0 and then let θ vary from 0 to 2π

$$i := \begin{cases} 0 & \text{if } FRAME \le 20 \\ (0..FRAME - 20) & \text{otherwise} \end{cases}$$

For the first 20 frames θ is fixed to be 0 then varies from 0 to 2 in increments of /24. This is standard for almost every case (as with polar coordinates).

$$\theta_i := \pi \cdot \frac{i}{24}$$

$$j := 0..10$$

This allows r to vary from 0 to 2 in increments of .1. The range on r is dependent on the particular problem.

$$r_j := \frac{j}{10}$$

Axes:

$$l := 0..20 \quad X1_{i,l} := 0 \quad Y1_{i,l} := 0 \quad Z1_{i,l} := (l - 10) \cdot .1$$

The parabaloid

$$X_{i,j} := (r_j) \cdot \cos(\theta_i) \quad Y_{i,j} := (r_j) \cdot \sin(\theta_i) \quad Z_{i,j} := \left[1 - (X_{i,j})^2 - (Y_{i,j})^2\right]$$

The following 6 sets of parametric equations give the line segments from the plane to the parabaloid for y = 0, creating the cross section

$$s := 0..FRAMI \quad t(s) := s \cdot .25 \quad w := 0..1$$

$$x_{w,s} := 0 \quad y_{w,s} := 0 \quad z_{w,s} := t(s)$$

$$x1_{w,s} := \begin{cases} .1 & \text{if } FRAME \le 10 \\ .2 & \text{otherwise} \end{cases} \qquad y1_{w,s} := 0$$

$$z1_{w,s} := \begin{cases} t(s) & \text{if } FRAME \le 10 \\ \frac{.96}{.5} \cdot (t(s) - 2.5) & \text{if } 10 < FRAME \le 12 \\ \frac{.96}{.5} \cdot [t(s) - 2.5 - .25 \cdot (FRAME - 12)] & \text{otherwise} \end{cases}$$

$$x2_{w,s} := \begin{cases} .1 & \text{if } FRAME \le 12 \\ .4 & \text{otherwise} \end{cases} \qquad y2_{w,s} := 0 \qquad z2_{w,s} := \left[\begin{cases} t(s) & \text{if } FRAME \le 12 \\ \frac{.84}{.5}\cdot(t(s) - 3) & \text{if } 12 < FRAME \le 14 \\ \frac{.84}{.5}\cdot[t(s) - 3 - .25\cdot(FRAME - 14)] & \text{otherwise} \end{cases} \right]$$

$$x3_{w,s} := \begin{cases} .1 & \text{if } FRAME \le 14 \\ .6 & \text{otherwise} \end{cases} \qquad y3_{w,s} := 0 \qquad z3_{w,s} := \left[\begin{cases} t(s) & \text{if } FRAME \le 14 \\ \frac{.64}{.5}\cdot(t(s) - 3.5) & \text{if } 14 < FRAME \le 16 \\ \frac{.64}{.5}\cdot[t(s) - 3.5 - .25\cdot(FRAME - 16)] & \text{otherwise} \end{cases} \right]$$

$$x4_{w,s} := \begin{cases} .1 & \text{if } FRAME \le 16 \\ .8 & \text{otherwise} \end{cases} \qquad y4_{w,s} := 0 \qquad z4_{w,s} := \left[\begin{cases} t(s) & \text{if } FRAME \le 16 \\ \frac{.36}{.5}\cdot(t(s) - 4) & \text{if } 16 < FRAME \le 18 \\ \frac{.36}{.5}\cdot[t(s) - 4 - .25\cdot(FRAME - 18)] & \text{otherwise} \end{cases} \right]$$

$$x5_{w,s} := \begin{cases} .1 & \text{if } FRAME \le 18 \\ .9 & \text{otherwise} \end{cases} \qquad y5_{w,s} := 0 \qquad z5_{w,s} := \left[\begin{cases} t(s) & \text{if } FRAME \le 18 \\ \frac{.19}{.5}\cdot(t(s) - 4.5) & \text{if } 18 < FRAME \le 20 \\ \frac{.19}{.5}\cdot[t(s) - 4.5 - .25\cdot(FRAME - 20)] & \text{otherwise} \end{cases} \right]$$

$(X, Y, Z), (Y1, Z1, X1), (Z1, X1, Y1), (x, y, z), (x1, y1, z1), (x2, y2, z2), (x3, y3, z3), (x4, y4,$