Find the surface Integral of $f(x,y,z) = \left(x^2 + y^2\right)\cdot z$ where σ is the portion of the sphere $x^2 + y^2 + z^2 = 4$ above the plane z = 1.

$$S = \int\int f(x,y,z)\cdot\sqrt{1 + \left(\frac{\delta z}{dx}\right)^2 + \left(\frac{\delta z}{dy}\right)^2}\,dy\,dx$$

a. $z = \sqrt{4 - x^2 - y^2}$

$$\frac{\delta z}{dx} = -\frac{x}{\sqrt{4 - x^2 - y^2}}$$

$$\frac{\delta z}{dy} = -\frac{y}{\sqrt{4 - x^2 - y^2}}$$

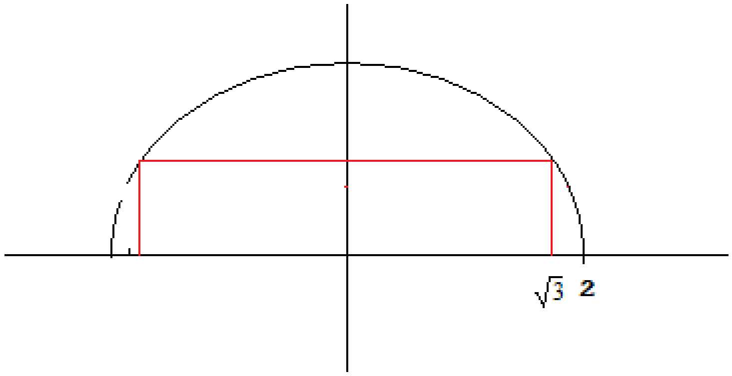
$$\sqrt{1 + \left(\frac{\delta z}{dx}\right)^2 + \left(\frac{\delta z}{dy}\right)^2} = \sqrt{1 + \left(\frac{-x}{\sqrt{4 - x^2 - y^2}}\right)^2 + \left(-\frac{y}{\sqrt{4 - x^2 - y^2}}\right)^2} = \frac{2}{\sqrt{4 - x^2 - y^2}} = \frac{2}{z}$$

$$f(x,y,z) = \left(x^2 + y^2\right)\cdot z$$

$$f(x,y,z)\cdot\sqrt{1 + \left(\frac{\delta z}{dx}\right)^2 + \left(\frac{\delta z}{dy}\right)^2} = \frac{2}{z}\left(x^2 + y^2\right)\cdot z = 2\left(x^2 + y^2\right)$$

In cylindrical coordinates we have simply 2r

Now to set up the region of integration



$$x^2 + y^2 + z^2 = 4 \qquad z = 1$$

We have $x^2 + y^2 = 3$

In polar coordinates we have simply $r = \sqrt{3}$ as the region of integration in the x- y plane

Finally we have :

$$\int_0^{2\cdot\pi} \int_0^{\sqrt{3}} 2r \cdot r \, dr \, d\theta$$