Summary -Observations and Terminology

The following are based on the animation f(x) and f ' (x) but apply to all differentiable functions in general.

Observation 1
f(x) is increasing on an open interval if and only if f ' (x) is positve on that interval

Note f is increasing on the intervals (0,1),(3,5) and (5,7) precisely the intervals where f ' (x) is positive.

Observation 2

f(x) is decreasing on an open interval if and only if f ' (x) is negative on that interval .

Note f is decrasing on the interval (3,5).

Observation 3

f(x) has a horizontal tangent line where f ' (x) = 0. f(x) has horizontal tangents at x = 1,3,and 5.

Terminology 1

If f ' (c) = 0 we say f(x) has a critical point at x = c. Another term for a critical point is stationary point.

Terminolgy 2

If $f(c) \geq f(x)$ for all x in some open interval containing c we say f(x) has a local maximum at x = c. f(c) is called the local maximum value.

Terminology 3

If $f(c) \leq f(x)$ for all x in some open interval containing c we say f(x) has a local minimum

at x = c. f(c) is called the local minimum value.

Terminolgy 4

If $f(c) \geq f(x)$ for all x then we say f(x) has an absolute or global maximum at x = c. Sometimes if we just say maximum we mean global or absolute maximum. f(c) is called the absolute maximum value.

Terminology 5

If f(c) $\leq$ f(x) for all x then we say f(x) has an absolute or global minimum at x = c.
Sometimes if we just say minimum we mean global or absolute minimum.
f(c) is called the absolute minimum value.

Terminology 6

Either a local minimum or local maximum is referred to as an extremum

singular	plural
maximum	maxima
minimum	minima
extremum	extrema

Observation 4

If f(x) is differentiable then the local extrema occur at the critical points . Note f(x) has a local maximum at the critical point x = 1 and a local minimum at the critical point x = 3.

Be Careful 1

Not every critical point is an extremum as is evidenced by f(x) at x = 5.

Observation 5

If f ' (x) changes from positive to negative at a critical point x = c then f(x) has a local maximum at x = c as is evidenced at x = 1.

Observation 6

If f ' (x) changes from negative to positive at a critical point x = c then f(x) has a local minimum at x = c as is evidenced at x = 3.

Observation 7

If f ' (x) doesn't change sign at a critical point x = c f(x) has an inflection point at x= c i.e. a

point where the concavity of f(x) changes.

Further Details

1. Extreme Value Theorem

If f(x) is continuous on the closed interval [a,b] it has both a maximum and a minimum.

Observation 8

If f(x) is continuous on [a,b] and differentiable on (a,b) then the extrema occur either at the endpoints or at critical points.

2. If f(x) is not differentiable then a local extremum can occur where the derivative does not exist.

For example for $f(x) = |x|$ f(x) is not differentiable at $x = 0$ but there is a local (in fact global) minimum at $x = 0$.

See the download A Non-Differentiable Example for the details

Terminology 7

If f(x) is continuous at $x = c$ but not differentiable we say f(x) has a cusp at $x = c$.

Two examples are $f(x) = |x|$ at $x = 0$ and $f(x) = x^{2/3}$ at $x = 0$.