Let $\vec{F} = 6 \cdot y \cdot x^2 \cdot \vec{i} + 2 \cdot x^3 \cdot \vec{j} + 6 \cdot x \cdot y \cdot \vec{k}$ and Let C be the curve of intersection of the

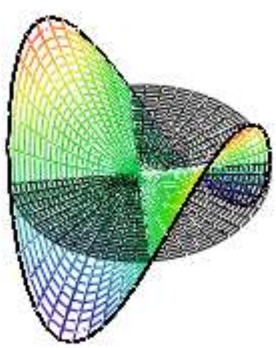
saddle: $z = y^2 - x^2$ and the cylinder $x^2 + y^2 = 25$.

a. Obtain a parameterization of the curve and compute $\int_a^b \vec{F} \cdot \frac{\overrightarrow{dr}}{dt} \, dt$

b. Use Stokes Theorem to evaluate the line integral.

This is one problem that if nothing else shows why you want Stokes Thm in your toolbox

The intersection of the saddle with the cylinder yields a curve which follows the contour of the saddle over a circle of radius 5 in the x-y plane



a. We begin by parameterizing the curve with:

$x = 5 \cdot \cos(t)$

$y = 5 \cdot \sin(t)$

$z = y^2 - x^2 = 25 \sin(t)^2 - 25 \cos(t)^2 = -25 \cos(2 \cdot t)$

$\vec{r} = 5 \cos(t) \vec{i} + 5 \cdot \sin(t) \cdot \vec{j} + (-25 \cos(2 \cdot t)) \cdot \vec{k}$

$$\frac{d\vec{r}}{dt} = -5\cdot\sin(t)\cdot\vec{i} + 5\cdot\cos(t)\cdot\vec{j} + 50\cdot\sin(2\cdot t)\cdot\vec{k}$$

$$\vec{F} = 6\cdot y\cdot x^2\cdot\vec{i} + 2\cdot x^3\cdot\vec{j} + 6\cdot x\cdot y\cdot\vec{k}$$

$$\vec{F} = 750\cdot\cos(t)^2\cdot\sin(t)\cdot\vec{i} + 250\cdot\cos(t)^3\cdot\vec{j} + 150\cdot\cos(t)\cdot\sin(t)\cdot\vec{k}$$

$$\vec{F}\cdot\frac{d\vec{r}}{dt} = -3.75\times 10^3\cos(t)^2\cdot\sin(t)^2 + \left(1.25\times 10^3\right)\cdot\cos(t)^4 + 1.5\,10^4\cdot\left(\cos(t)^2\cdot\sin(t)^2\right)$$

$$\vec{F}\cdot\frac{d\vec{r}}{dt} = 1.25\,10^3\cdot\cos(t)^4 + 1.125\,10^4\cos(t)^2\cdot\sin(t)^2$$

$$\int_0^{2\cdot\pi} 1250\cos(t)^4 + 11250\cos(t)^2\cdot\sin(t)^2\,dt \rightarrow 3750\pi$$

b. Using Stoke's Theorem:

$$\nabla\times F = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ 6\cdot y\cdot x^2 & 2\cdot x^3 & 6\cdot x\cdot y \end{pmatrix} = 6\cdot x\cdot\vec{i} - 6\cdot y\cdot j$$

$$\vec{N} = 2\cdot x\cdot\vec{i} - 2\cdot y\cdot\vec{j} + \vec{k}$$

$$(\nabla\times F)\cdot\vec{N} = 12\cdot x^2 + 12\cdot y^2$$

$$\int_{0}^{2\cdot\pi}\int_{0}^{5} 12\cdot r^{2}\cdot r\,dr\,d\theta = \int_{0}^{2\cdot\pi}\int_{0}^{5} 12\cdot r^{3}\,dr\,d\theta = 3750\pi$$