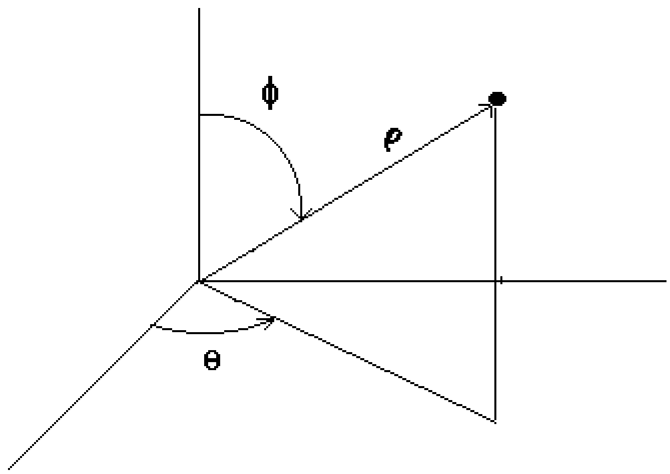
Lab 4 Spherical Coordinates

In the last lab we considered the cylindrical coordinate system. In this lab we consider the spherical coordinate system.

We describe a point in 3 space by the coordinates ρ, θ, and ϕ where ρ is the distance from the origin to the point, θ is the polar angle measured counterclockwise from the positive x axis (as in polar coordinates), and ϕ is the azimuthal angle i.e. the angle measured from the positive z axis.



We have the following ranges on the variables:

$0 \leq \rho < \infty$

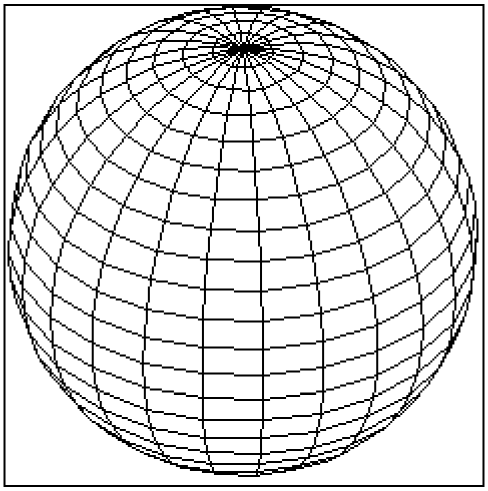
$0 \leq \theta < 2\pi$

$0 \leq \phi < \pi$

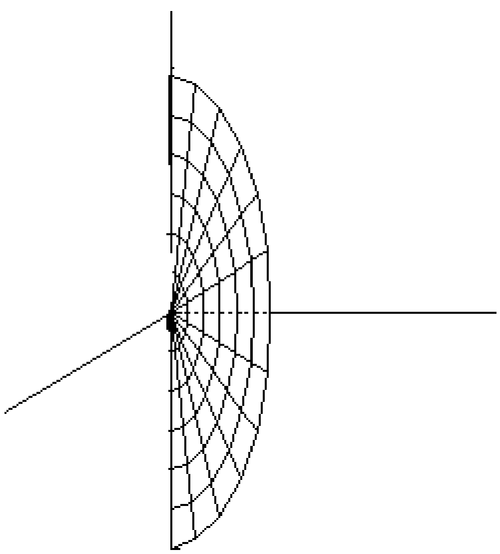
Functions

Functions in spherical coordinates are of the form $\rho = f(\theta,\phi)$.

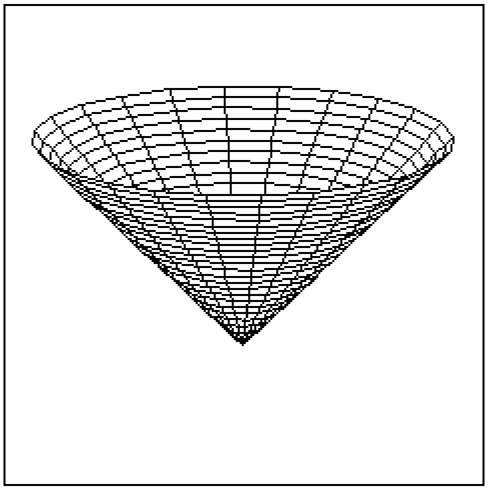
Constant functions and relations



ρ = constant - A sphere i.e. the set of all points
equidistant from the origin



θ = constant - A vertical plane as in the case of cylindrical coordinates.



ϕ = constant - A cone is generated as the azimuthal angle does not change.

Conversion to rectangular coordinates

As in the case of cylindrical coordinates we define ρ as a function of θ and ϕ then convert to rectangular coordinates and create a parametric surface plot.

The conversions are :

$$x = \rho\sin(\phi)\cos(\theta)$$
$$y = \rho\sin(\phi)\sin(\theta)$$
$$z = \rho\cos(\phi)$$

Typically $\rho = \rho(\theta,\phi)$

Formatting the computer

$i := 0..\,48 \quad \theta_i := \pi\cdot\frac{i}{24}$ as in the case of cylindrical coordinates

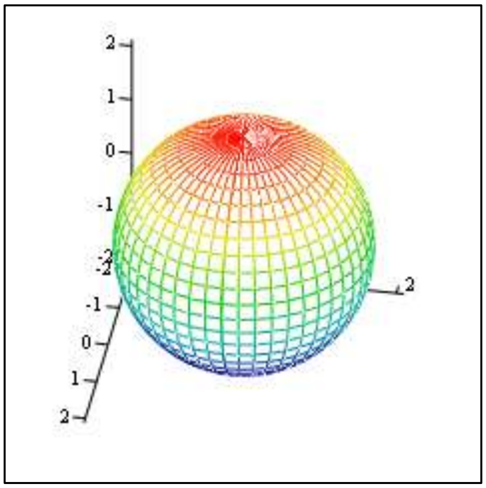
$j := 0..\,24 \quad \phi_j := \pi\cdot\frac{j}{24}$ similar to θ , however j only goes to 24 since ϕ only goes to π.

Now is where we would define ρ . For example to graph the simple sphere $\rho = 2$

$\rho_{i,j} := 2$

$X_{i,j} := \rho_{i,j}\cdot\cos(\theta_i)\cdot\sin(\phi_j) \quad Y_{i,j} := \rho_{i,j}\cdot\sin(\theta_i)\cdot\sin(\phi_j) \quad Z_{i,j} := \rho_{i,j}\cdot\cos(\phi_j)$ these are the

conversions to rectangular coordinates.



(X, Y, Z)

I formatted this as a color map with HIDE LINES turned on

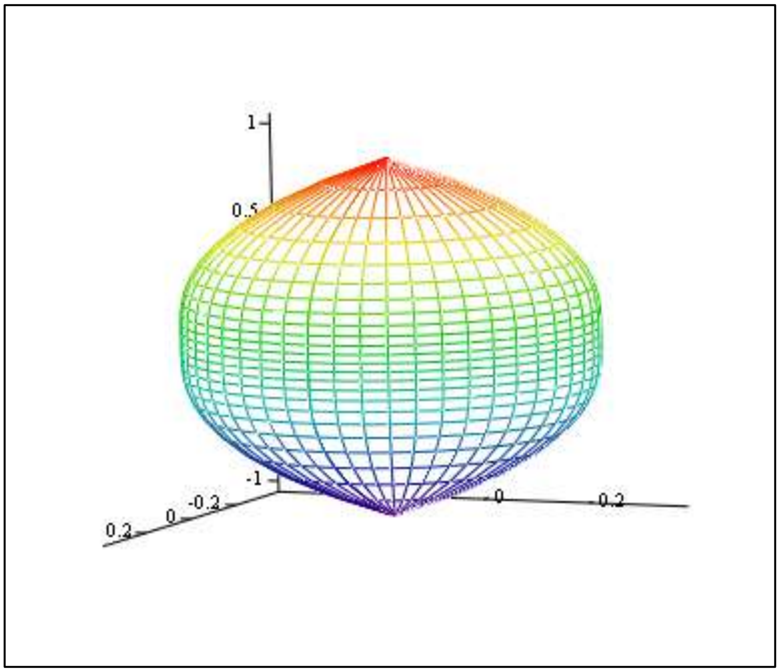
Let's Consider one more example. Let's take $\rho = e^{-\sin(\phi_j)}$

$i := 0..48 \quad \theta_i := \pi \cdot \frac{i}{24}$ as before

$j := 0..24 \quad \phi_j := \pi \cdot \frac{j}{24}$ as before

$\rho_{i,j} := e^{-\sin(\phi_j)}$ Note this is the only difference because we have a new function Notice we have to subscript j

$X_{i,j} := \rho_{i,j} \cdot \cos(\theta_i) \cdot \sin(\phi_j) \quad Y_{i,j} := \rho_{i,j} \cdot \sin(\theta_i) \cdot \sin(\phi_j) \quad Z_{i,j} := \rho_{i,j} \cdot \cos(\phi_j)$ same as before



(X, Y, Z)

Exercises
Graph the following Make sure you subscript ϕ and θ with j and i respectively

1. $\rho = 1 - \cos(\phi)$ (an apple)

2. $\rho = [1-\cos(\phi)][\ 8 + |\sin(7\theta)|\ ]$ (making the apple in 1 into a pumpkin by using a sinusoidally varying radius)

3. $\rho = \cos(2\phi)$ should look something like a p orbital---recall in polar coordinates $r = \cos(2\theta)$ is a

4 - petaled rose-- spin it around.