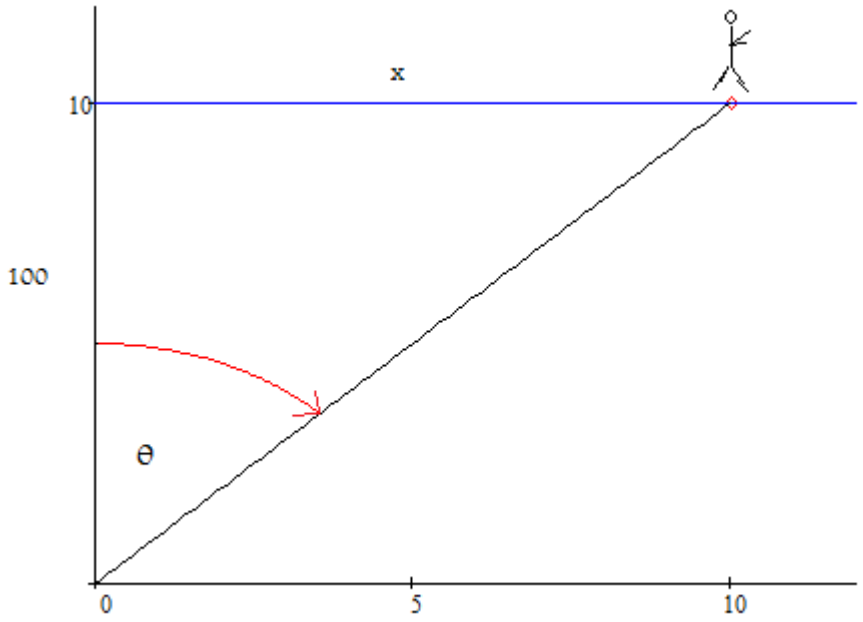
Running Down the Beach

A person is running down Vilano Beach. A St John's county sheriff's deputy is 100 ft from the beach and has a spotlight on the person as he runs along the straight shoreline. If the spotlight is rotating at 0.2 rads/sec when our hero is 100 ft down the beach how fast is he running?

See the animation-Running Down the Beach



$$\tan(\theta) = \frac{x}{100} = .01 \cdot x$$

Note some people might be tempted to use the sine or cosine function. The reason we don't is that we would have to use s- the distance from the sheriff's deputy to the person and we don't know ds/dt or dx/dt so we would have 2 unknown quantities. similarly you might be tempted to use the Pythagorean theorem but this would also fail for the same reason.

$$\sec^2(\theta) \cdot \frac{d\theta}{dt} = .01 \cdot \frac{dx}{dt}$$

$$\sec^2(\theta) \cdot .2 = .01 \cdot \frac{dx}{dt}$$

When the person is 100ft down the beach $\sec(\theta) = \sqrt{2}$.

$$2 \cdot .2 = .01 \cdot \frac{dx}{dt}$$

$\frac{dx}{dt} = 40 \frac{ft}{s}$ which is a little over 27 mph-- what was our hero hiding?