Formatting Animations for Riemann Sums

Right Sums

In the example on the site we used Riemann Sums to approximate $\int_1^2 x^2\,dx$

$x := 1, 1.001..2$ Sets up the interval along the x -axis

$f(x) := x^2$ The function in question

$n := 2 + FRAME$ sets up the number of subintervals. I use n+2 so the initial image already has 2 rectangles

We'll need the following information:

1. The width of each sub interval $\Delta x = \dfrac{b-a}{n}$ In our example $\Delta x = \dfrac{1}{n}$

2, Define the partition points $x_k = a + \dfrac{k\cdot(b-a)}{n}$ In our example $x_k = 1 + \dfrac{k}{n}$

3. Compute $f(x_k)$ the height of each rectangle over each sub interval $f(x_k) = \left(1 + \dfrac{k}{n}\right)^2$

4. The right Riemann sum -- $\sum_{k=1}^{n} \left(f(x_k)\cdot\Delta x\right)$ In our example

$$\sum_{k=1}^{n} \left[\left(1 + \frac{k}{n}\right)^2 \cdot \frac{1}{n}\right]$$

5. Define $fR(x) = f(a + \Delta x) + \sum_{k=1}^{n} \left[\Phi(x - x_k)\cdot\left(f(x_{k+1}) - f(x_k)\right)\right]$ to graph the rectangles

$$fR(x) := \left(1 + \frac{1}{n}\right)^2 + \sum_{k=1}^{n} \left[\Phi\left[x - \left(1 + \frac{k}{n}\right)\right]\cdot\left[\left(1 + \frac{k+1}{n}\right)^2 - \left(1 + \frac{k}{n}\right)^2\right]\right]$$

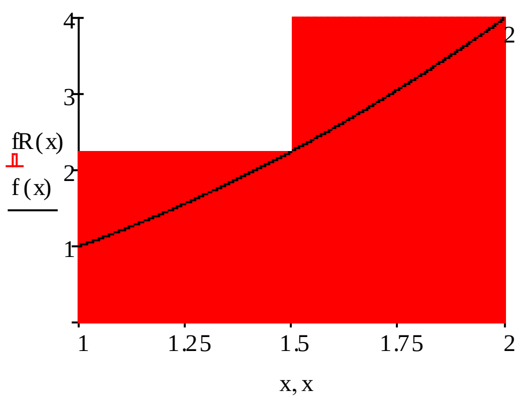
The actual format looks like:

$x := 1, 1.001 .. 2$ $\quad n := 2 + FRAME$ $\quad f(x) := x^2$

$$fR(x) := \left(1 + \frac{1}{n}\right)^2 + \sum_{k=1}^{n} \left[\Phi\left[x - \left(1 + \frac{k}{n}\right)\right] \cdot \left[\left(1 + \frac{k+1}{n}\right)^2 - \left(1 + \frac{k}{n}\right)^2\right]\right]$$

<u>On the Graph for Trace 1 change from lines to bars under TYPE menu</u>

Rn = $\sum_{k=1}^{n} \left[\left(1 + \frac{k}{n}\right)^2 \cdot \frac{1}{n}\right] = 3.125$



The Left sum is almost identical. The only differences are:

1. In $fL(x) = f(a) + \sum_{k=1}^{n} \left[\Phi\left(x - x_{k+1}\right) \cdot \left(f\left(x_{k+1}\right) - f\left(x_k\right)\right)\right]$

i.e. we use f (a) instead of f(a+Δ x), sum from 0 to n-1 instead of 1 to n and replace $\Phi\left(x - x_k\right)$ with $\Phi\left(x - x_{k+1}\right)$

2. For Ln = $\sum_{k=0}^{n-1} \left[\left(1 + \frac{k}{n}\right)^2 \cdot \frac{1}{n}\right]$ we again sum from 0 to n-1

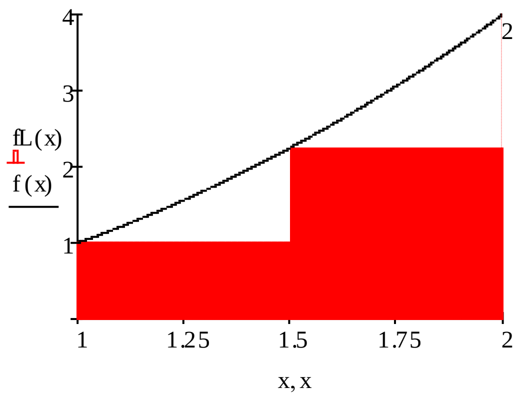
$$x := 1, 1.001 .. 2$$

$$n := 2 + \text{FRAMI}$$

$$f(x) := x^2$$

$$fL(x) := (1)^2 + \sum_{k=0}^{n-1} \left[\Phi\left[x - \left(1 + \frac{k+1}{n}\right)\right]\right] \cdot \left[\left(1 + \frac{k+1}{n}\right)^2 - \left(1 + \frac{k}{n}\right)^2 \right]$$

$$Ln = \sum_{k=0}^{n-1} \left[\left(1 + \frac{k}{n}\right)^2 \cdot \frac{1}{n} \right] = 1.625$$



The Midpoint Rule

The midpoint of each subinterval is $c_k = \dfrac{x_k + x_{k-1}}{2}$

In our example

$$c_k = \frac{1 + \frac{k}{n} + \left(1 + \frac{k-1}{n}\right)}{2} = \frac{2k + 2n - 1}{2 \cdot n}$$

$$\text{Mn} = \sum_{k=1}^{n} \left(f\left(c_k\right) \cdot \Delta x\right) = \sum_{k=1}^{n} \left[\left(\frac{2k + 2n - 1}{2 \cdot n}\right)^2 \cdot \frac{1}{n}\right]$$

$$\text{fM}(x) = \left(1 + \frac{\Delta x}{2}\right) + \sum_{k=1}^{n} \left[\Phi\left(x - x_k\right) \cdot \left(f\left(c_{k+1}\right) - f\left(c_k\right)\right)\right]$$

$$\text{fM}(x) := \left(1 + \frac{1}{2n}\right)^2 + \sum_{k=1}^{n} \left[\Phi\left[x - \left(1 + \frac{k}{n}\right)\right] \cdot \left[\left[\frac{2(k+1) + 2n - 1}{2 \cdot n}\right]^2 - \left(\frac{2k + 2n - 1}{2 \cdot n}\right)^2\right]\right]$$

$$\text{Mn} = \sum_{k=1}^{n} \left[\left(\frac{2k + 2n - 1}{2 \cdot n}\right)^2 \cdot \frac{1}{n}\right] = 2.313$$