Using Power Series show the solution to

dy/dx + x = y is $y = 1 + x + Ce^x$.

Verify this result using an appropriate integrating factor

$$\frac{dy}{dx} - y + x = 0$$

$$y = \sum_{n=0}^{\infty} \left(C_n \cdot x^n\right)$$

$$\frac{dy}{dx} = \sum_{n=1}^{\infty} \left(n \cdot C_n \cdot x^{n-1}\right) = \sum_{n=0}^{\infty} \left[(n+1) \cdot C_{n+1} \cdot x^n\right]$$

$$\sum_{n=0}^{\infty} \left[(n+1) \cdot C_{n+1} \cdot x^n\right] - \left[\sum_{n=0}^{\infty} \left(C_n \cdot x^n\right)\right] + x = 0 \qquad (1)$$

Consider the coefficients

n = 0 $C_1 - C_0 = 0$

For n=1 we are considering the coefficients of the x terms so include 1 from the + x in eqn (1) .

$$2 \cdot C_2 - C_1 + 1 = 0$$

$$C_2 = \frac{C_1 - 1}{2} = \frac{C_0 - 1}{2}$$

It follows then

$$C_n = \frac{C_0 - 1}{n!}$$

$$y = C_0 + C_0 \cdot x + \sum_{n=2}^{\infty} \left[\frac{(C_0 - 1)}{n!} \cdot x^n \right]$$

$$y = C_0 - 1 + 1 + C_0 \cdot x - x + x + \sum_{n=2}^{\infty} \left[\frac{(C_0 - 1)}{n!} \cdot x^n \right]$$

$$y = 1 + x + \sum_{n=0}^{\infty} \left[\frac{(C_0 - 1)}{n!} \cdot x^n \right]$$

$$y = 1 + x + (1 - C_0) \cdot e^x = 1 + x + C \cdot e^x$$

To verify we write:

$$\frac{dy}{dx} - y = -x$$

The integrating factor is e^{-x}

$$\frac{d(e^{-x} \cdot y)}{dx} = -x \cdot e^{-x}$$

$$e^{-x} \cdot y = x \cdot e^{-x} + e^{-x} + C$$

$$y = x + 1 + C \cdot e^x$$