The Damped Harmonic Oscillator

If a mass is initially compressed one unit and released in the absence of air resistance then the position is given by y(t) = cos(t). See Animation No Damping.

It is easily seen that the maxima and minima occur at multiples of π since

$$\frac{d(\cos(t))}{dt} = -\sin(t).$$

However a more realistic situation involves air resistance in which the amplitude decays to zero. In this case we say the motion is damped. See animation Damping.

As you will learn in differential equations (or if you want see my DiffEq page -2d Order Diff Eqs-Motion) typically the position is a product of a decaying exponential and a trig function

A typical example would be y(t) = $e^{-.1\cdot t}\cdot\cos(1.3t)$.

Where do the maxima and minima occur now?

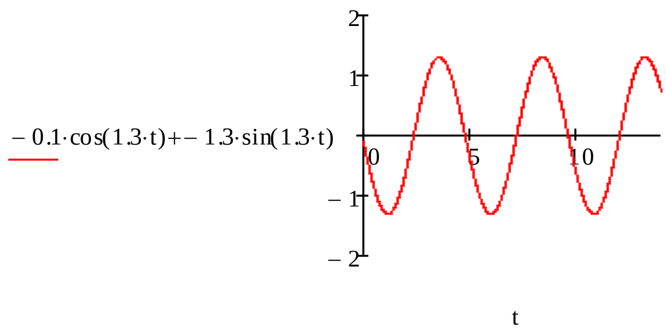
Using the product rule:

y(t) = $e^{-.1\cdot t}\cdot\cos(1.3t)$

by differentiation, yields

$$\frac{dy}{dt} = -0.1\cdot\cos(1.3t)\cdot e^{-0.1\cdot t} + -1.3\sin(1.3t)\cdot e^{-0.1\cdot t}$$

The maxima and minima occur when $-0.1\cdot\cos(1.3t) + -1.3\sin(1.3t) = 0$



Solving this graphically I obtain minima at 2.32 and 7.182

and maxima at 4.788 and 9.604.

Using a Mathcad Solve block we have

Given

$$-0.1\cdot\cos(1.3t) + -1.3\sin(1.3t) = 0$$

$$t := \begin{pmatrix} 2.32 \\ 4.788 \\ 7.182 \\ 9.604 \end{pmatrix}$$ where our initial guesses are based on our graphical estimates

$$\text{Find}(t) = \begin{pmatrix} 2.358 \\ 4.774 \\ 7.191 \\ 9.607 \end{pmatrix}$$

The distance between successive minima is $7.191 - 2.358 = 4.833$

The distance between successive maxima is $9.607 - 4.774 = 4.833$

Note this is no surprise since $\frac{2\cdot\pi}{1.3} = 4.833$ (Recall from trig the period is $2\pi/b$).

Note the intervals between successive maxima or successive minima is 4.833, however the motion

is not truly periodic since the maxima have different values as do the minima. In this case we say the motion is quasi-periodic and 4.833 is the quasi period.