Creating Animations in Polar Coordinates and Tangent Lines

Creating animations in polar coordinates is very similar to creating animations in rectangular coordinates.

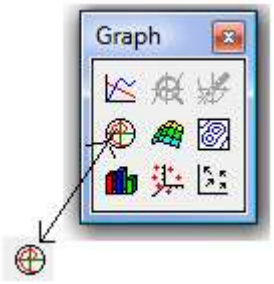
1. Define the range on the independent variable

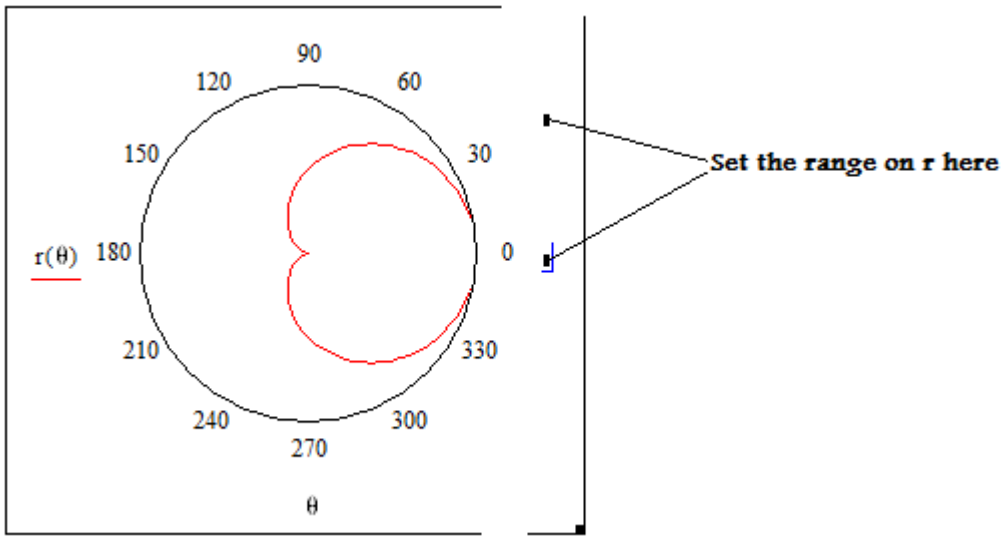
$\theta := 0, \frac{\pi}{48} .. 2\cdot\pi$ here we use a step size of $\frac{\pi}{48}$ so we can see how the graph generates in multiples of π. To animate we'll change 2π to $\frac{\pi}{48}\cdot FRAME$

2. Define the function to be graphed. For example suppose we want to graph the cardioid:

$r(\theta) := 1 + \cos(\theta)$

From the graphing window click the polar graph icon.



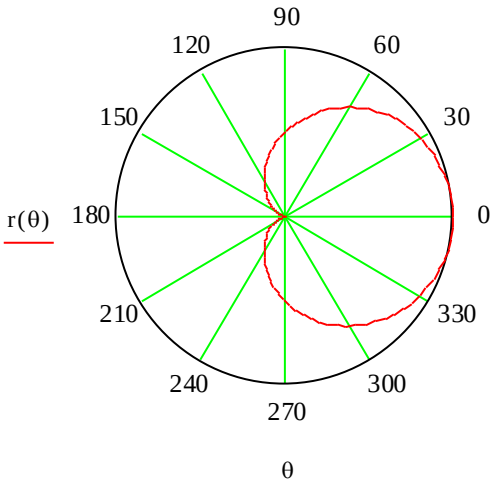


Since for r = 1+cos(θ) the maximum value of r is 2 I put 0 and 2.1 in the placeholder.

Of course the way you choose to format your graph is up to you but I'll make the following changes

1. Under the tab Polar Axes under Radial I turn numbered off.

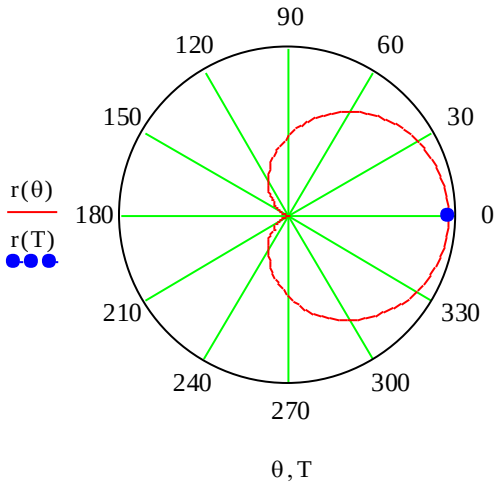
2. Under Angular I turn grid lines on.



Now we'll add a point to generate the curve by defining :

$T := \frac{FRAME}{48}$ and putting an extra copy of r(T) and T on the graph.

Under Traces change Trace 2 to points and add a symbol.



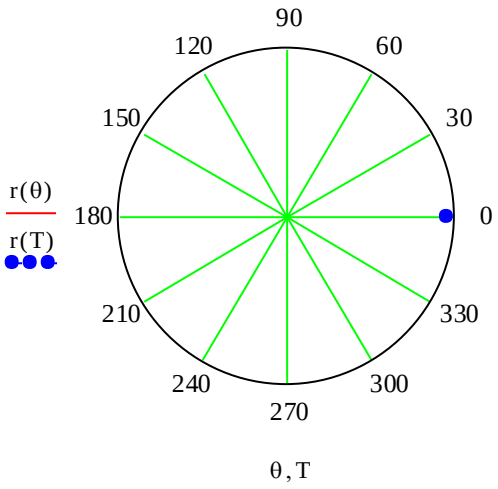
Now we are ready to animate:

$$\theta := 0, \frac{\pi}{48} .. \frac{\pi}{48} \cdot \text{FRAME}$$

$$r(\theta) := 1 + \cos(\theta)$$

$$T := \frac{\pi}{48} \cdot \text{FRAME}$$

$T \to 0$ $\quad$ $r(T) = 2$



Now we add the tangent line --See Calculus7.com for the development of the formulas.

$$r = \frac{-\left[m_{tan} \cdot \left(r\left(\theta_0\right) \cdot \cos\left(\theta_0\right)\right)\right] + r\left(\theta_0\right) \cdot \sin\left(\theta_0\right)}{\left(\sin(\theta) - m_{tan} \cdot \cos(\theta)\right)}$$

where $m_{tan} = \dfrac{\frac{dr}{d\theta} \cdot \sin\left(\theta_0\right) + r \cdot \cos\left(\theta_0\right)}{\frac{dr}{d\theta} \cdot \cos\left(\theta_0\right) - r \cdot \sin\left(\theta_0\right)}$

For example with our Cardioid the polar form of the tangent line we have:

$$m_{tan} = \frac{2\cdot\cos^2(\theta) + \cos(\theta) - 1}{-2\sin(\theta)\cdot\cos(\theta) - \sin(\theta)}$$

$$m(\theta) := \frac{2\cdot\cos(\theta)^2 + \cos(\theta) - 1}{-2\sin(\theta)\cdot\cos(\theta) - \sin(\theta)}$$

$$r2(\theta 2) := \frac{-m(\theta)\cdot(1 + \cos(\theta))\cdot\cos(\theta) + (1 + \cos(\theta))\cdot\sin(\theta)}{\sin(\theta 2) - m(\theta)\cdot\cos(\theta 2)}$$

We also want to add a line segment from the origin to each point on the curve as it is being generated:

$s := 0, .1..1 \quad r1(s) := (1 + \cos(\theta))\cdot s$

The Set up is

$\theta := \frac{\pi}{24}\cdot FRAME$ defines θ at a single point.

$\theta 1 := 0, \frac{\pi}{24}..\frac{\pi}{24}\cdot FRAME$ defines the range of θ as the curve is generated.

$\theta 2 := 0, .1..2\cdot\pi$ defines the range variable for the tangent line

$r(\theta 1) := 1 + \cos(\theta 1)$

$$m(\theta) := \frac{2\cdot\cos(\theta)^2 + \cos(\theta) - 1}{-2\sin(\theta)\cdot\cos(\theta) - \sin(\theta)}$$

$$r2(\theta 2) := \frac{-m(\theta)\cdot(1 + \cos(\theta))\cdot\cos(\theta) + (1 + \cos(\theta))\cdot\sin(\theta)}{\sin(\theta 2) - m(\theta)\cdot\cos(\theta 2)}$$

$s := 0, .1..1 \quad r1(s) := (1 + \cos(\theta))\cdot s$

In the place holders for r set the values -2.1 and 2.1 Use 48 Frames.

$\theta \rightarrow 0$ $m(\theta) =$