Riemann Sums in Polar Coordinates

Here we'll develop the Riemann Sum for approximating the area inside the cardioid r = 1 +cos(θ)

For $0 \leq \theta \leq \pi/3$

Set the number of circular sectors $n := 2 + \text{FRAMI}$

$f(\theta) := 1 + \cos(\theta)$ defines the function we want

$\Delta\theta = \frac{\pi}{3 \cdot n}$ divides the interval 0 to π /3 into n subintervals

$\theta_k = \frac{k \cdot \pi}{3 \cdot n}$ determines the partition points to evaluate f(θ)

$$f\left(\theta_k\right) = 1 + \cos\left(\frac{k \cdot \pi}{3 \cdot n}\right)$$

The Riemann Sum $A_{approx}(n) := \sum_{k=1}^{n} \left(\frac{1}{2} \cdot f\left(\theta_k\right)^2 \cdot \Delta\theta\right)$

In our example $A_{approx}(n) := \frac{\pi}{6} \cdot \sum_{k=1}^{n} \left[\frac{1}{n} \cdot \left(1 + \cos\left(\frac{k \cdot \pi}{3 \cdot n}\right)\right)^2\right]$

Setting up the graph for n circular sectors:

$$f2(\theta) := 1 + 1 \cdot \cos\left(\frac{\pi}{3 \cdot n}\right) + \sum_{k=1}^{n} \left[\Phi\left[\theta - \frac{(k) \cdot \pi}{3 \cdot n}\right] \cdot \left[1 + \cos\left[\frac{(k+1) \cdot \pi}{3 \cdot n}\right] - \left[1 + \cos\left[\frac{(k) \cdot \pi}{3 \cdot n}\right]\right]\right]\right]$$

$\theta := 0, \frac{\pi}{384} .. \frac{\pi}{3}$ Sets up the Interval to graph the curve 1 +cos(θ).

Why $\frac{\pi}{384}$? On the Graph We put f2(θ) on twice.
For the first under Trace 2 change points to solid and change the line weight to 3 and the color to black. This outlines the circular sectors in black.

For Trace 3 change from lines to bars--this colors in the sectors.

I started with $\frac{\pi}{48}$ but this didn't fill in well. Purely by experimenting I kept doubling 48 until the sectors filled in 48-96-192-384. (and they say math is exact, precise and only has one answer)

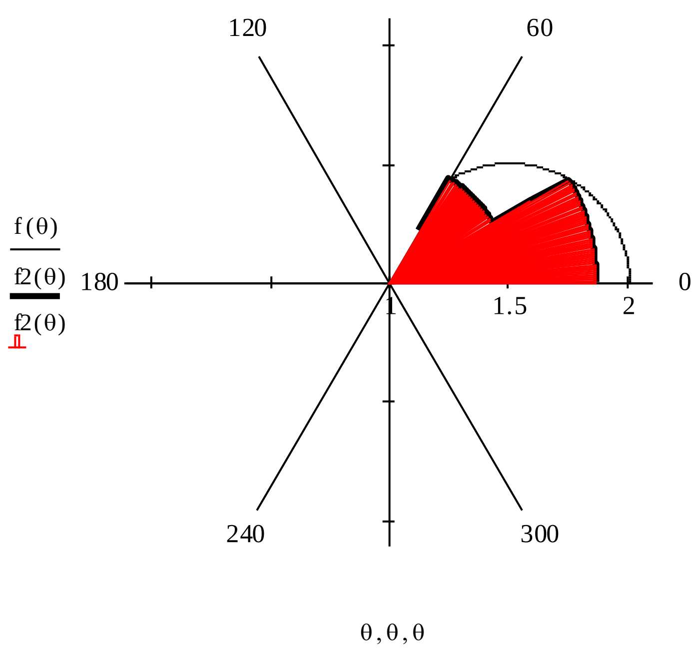
The Actual Format is:

$n := 2 + \text{FRAMI}$ $\quad \theta := 0, \frac{\pi}{384} .. \frac{\pi}{3}$ $\quad f(\theta) := 1 + \cos(\theta)$

$$f2(\theta) := 1 + 1 \cdot \cos\left(\frac{\pi}{3 \cdot n}\right) + \sum_{k=1}^{n} \left[\Phi\left[\theta - \frac{(k) \cdot \pi}{3 \cdot n}\right] \cdot \left[1 + \cos\left[\frac{(k+1) \cdot \pi}{3 \cdot n}\right] - \left[1 + \cos\left[\frac{(k) \cdot \pi}{3 \cdot n}\right]\right]\right]\right]$$

$$A(n) := \frac{\pi}{6} \cdot \sum_{k=1}^{n} \left[\frac{1}{n} \cdot \left(1 + \cos\left(\frac{k \cdot \pi}{3 \cdot n}\right)\right)\right]^2$$

$n = 2$ $\quad \text{AExact} = 1.7\text{(}$ $\quad A(n) = 1.501$



Now Animate. I used 30 Frames at 1 frame/sec