

Partial Derivatives

To illustrate we'll consider the partial derivatives of the paraboloid $f(x, y) = 1 - x^2 - y^2$ at the point $(1/3, 1/3, 7/9)$.

Our first set up will be for $\delta f / \delta y$

We start by defining the surface using the cylindrical coordinate format:

$$i := 0..48 \quad \theta_i := \pi \cdot \frac{i}{24} \quad j := 0..10 \quad r_j := \frac{j}{10}$$

$$X_{i,j} := r_j \cdot \cos(\theta_i) \quad Y_{i,j} := r_j \cdot \sin(\theta_i) \quad Z_{i,j} := 1 - (X_{i,j})^2 - (Y_{i,j})^2$$

For Plot 1 under the Appearance Tab change the weight to 0.1

Now we define the point

$$\tau := 0..1$$

$$a_{1,\tau} := \frac{1}{3} \quad b_{1,\tau} := \frac{1}{3} \quad c_{1,\tau} := \frac{7}{9}$$

For Plot 2 under the General Tab change from surface plot to Data Points and under the Appearance Tab change the weight to 3

Now we define the curve of intersection of the surface with plane $x = 1/3$

$$m := 0..100 \quad w(m) := .01 \cdot m$$

$$x_{\tau,m} := \frac{1}{3} \quad y_{\tau,m} := w(m) \quad z_{\tau,m} := 1 - (x_{\tau,m})^2 - (y_{\tau,m})^2$$

For Plot 3 under the Appearance Tab change the weight to 2.

Now we define the secant line through $(1/3, 1/3, 7/9)$.

$$h := .5 - .02 \cdot \text{FRAM}$$

(h is the interval over which the average rate of change is calculated as in the case of the secant line for functions of a single variable)

We can use $x_{1,m}$ and $y_{1,m}$ from the curve of intersection all we have to do is to define a new z value

$$\frac{\delta f}{\delta y} = -2 \cdot y \quad \text{so when } y=1/3 \text{ we have } \frac{\delta f}{\delta y} = \frac{-2}{3} \quad \text{so} \quad z = \frac{\delta f}{\delta y} \left(\frac{1}{3}, \frac{1}{3} + h \right) \left(y - \frac{1}{3} \right) + \frac{7}{9}$$

We define:

$$z_{\tau, m} := \left(\frac{-2}{3} - h \right) \cdot \left(\left(y_{\tau, m} - \frac{1}{3} \right) \right) + \frac{7}{9}$$

For Plot 4 under the Appearance Tab change the weight to 2.

The actual format is then :

$$i := 0..48 \quad \theta_i := \pi \cdot \frac{i}{24} \quad j := 0..10 \quad r_j := \frac{j}{10}$$

$$X_{i,j} := r_j \cdot \cos(\theta_i) \quad Y_{i,j} := r_j \cdot \sin(\theta_i) \quad Z_{i,j} := 1 - (X_{i,j})^2 - (Y_{i,j})^2$$

$$\tau := 0..1$$

$$a_{i,\tau} := \frac{1}{3} \quad b_{i,\tau} := \frac{1}{3} \quad c_{i,\tau} := \frac{7}{9}$$

$$m := 0..100 \quad w(m) := .01 \cdot m$$

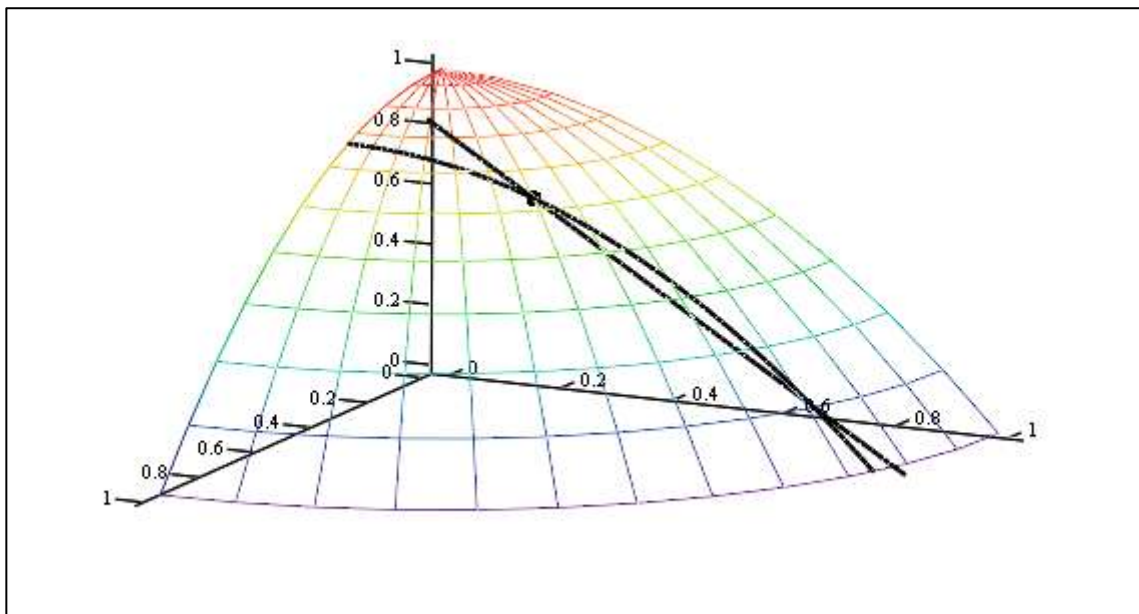
$$x_{\tau, m} := \frac{1}{3} \quad y_{\tau, m} := w(m) \quad z_{\tau, m} := 1 - (x_{\tau, m})^2 - (y_{\tau, m})^2$$

$$h := .5 - .02 \cdot \text{FRAM}$$

$$z_{\tau, m} := \left(\frac{-2}{3} - h \right) \cdot \left(\left(y_{\tau, m} - \frac{1}{3} \right) \right) + \frac{7}{9}$$

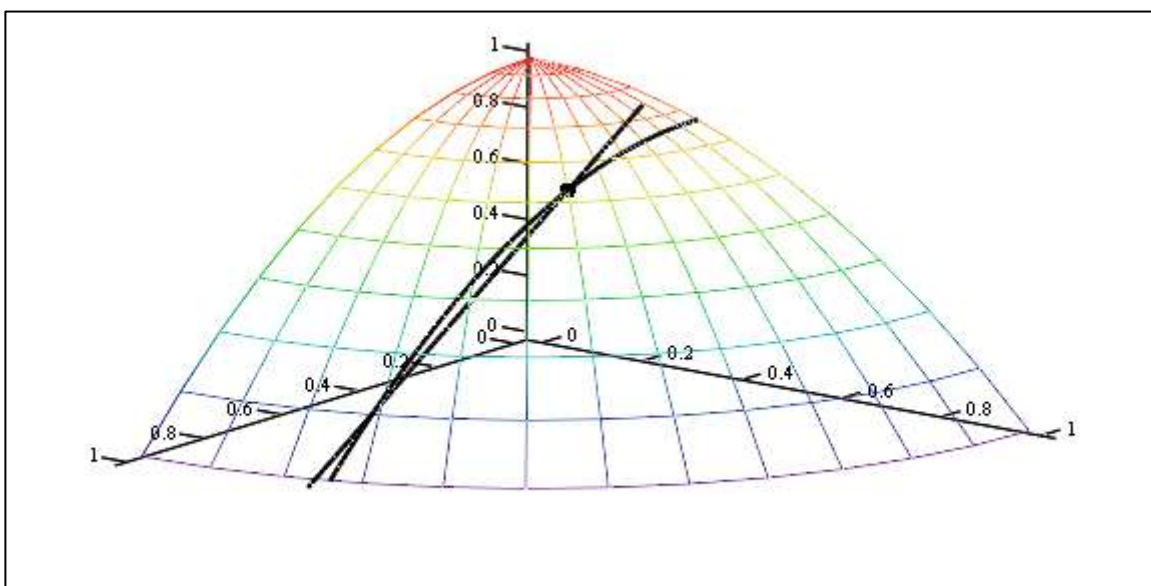
Use 24 frames.

| | Estimate | Exact |
|-----------|------------------------------------------|----------------|
| $h = 0.5$ | $\left(\frac{-2}{3} - h\right) = -1.167$ | $\frac{-2}{3}$ |



$(X, Y, Z), (a, b, c), (x, y, z), (x, y, z1)$

For the partial derivative with respect to x all we have to do is interchange x and y



$(X, Y, Z), (a, b, c), (y, x, z), (y, x, z1)$