

Show that the tangent lines to the hyperbolas $x^2 - y^2 = k$ and $xy = p$ are perpendicular at each point of intersection for all non-zero values of p and q . (For a complete discussion of orthogonal trajectories see my notes and animations on orthogonal trajectories on the differential equation page)

For the first hyperbola:

$$x^2 - y^2 = k$$

$$2x - 2y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{x}{y}$$

For the second hyperbola:

$$xy = p$$

$$x = \frac{p}{y}$$

$$1 = p \cdot \frac{-1}{y^2} \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = xy \cdot \frac{-1}{y^2} = \frac{-x}{y}$$

Since the slopes are negative reciprocals the curves are orthogonal-Below is the graph for $k=1, p=2$

