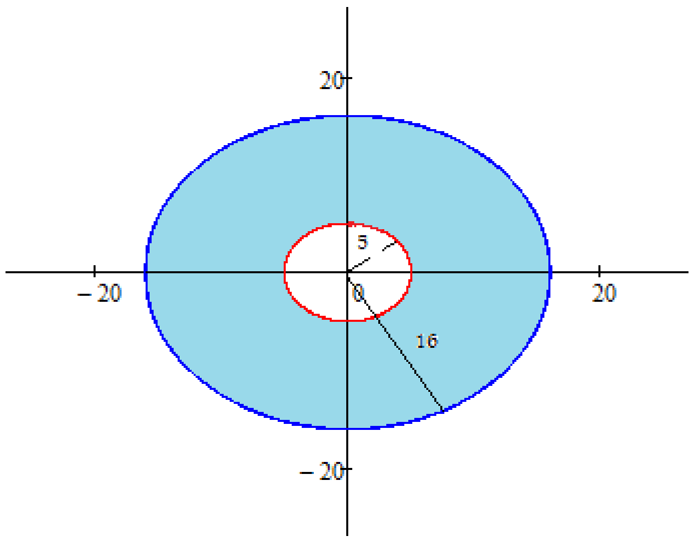
Suppose one circle has a radius which is increasing by 2cm/s.   It is inside and concentric with a second circle which is expanding at a rate of 1 cm/sec.
See Animation Expanding Circles.

1. At what rate is the Area between the 2 changing when the radius of the first circle is 5 cm and the radius of the second circle is 16.

2. At what rate is the Area between the 2 changing when the radius of the first circle is 13 cm and the radius of the second circle is 20.

3. Is the rate of change of the Area ever 0?



1.

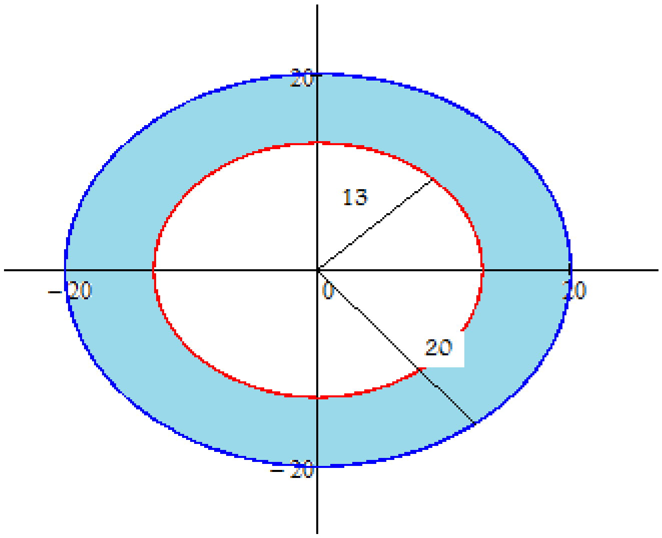
$$A = \pi \cdot (r2)^2 - \pi\, r1^2$$

$$\frac{dA}{dt} = 2 \cdot \pi \cdot r2 \cdot \frac{dr2}{dt} - 2 \cdot \pi \cdot r1 \cdot \frac{dr1}{dt}$$ Note $\frac{dA}{dt}$ depends on the radii and the rates of change of the radii

$$\frac{dA}{dt} = 2 \cdot \pi \cdot 16 \cdot 1 - 2 \cdot \pi \cdot 5 \cdot 2 = 37.69$$

Here the Area is increasing

2.



$$A = \pi \cdot (r2)^2 - \pi\, r1^2$$

$$\frac{dA}{dt} = 2 \cdot \pi \cdot r2 \cdot \frac{dr2}{dt} - 2 \cdot \pi \cdot r1 \cdot \frac{dr1}{dt}$$

$$\frac{dA}{dt} = 2 \cdot \pi \cdot 20 \cdot 1 - 2 \cdot \pi \cdot 13 \cdot 2 = -37.69$$

Here the radius is decreasing

3. $2 \cdot \pi \cdot r2 \cdot 1 - 2 \cdot \pi \cdot r1 \cdot 2 = 0$

$$r2 = 2 \cdot r1$$

We know r1 = 5 + 2t and r2 = 13 + t

$$10 + 4 \cdot t = 13 + t$$

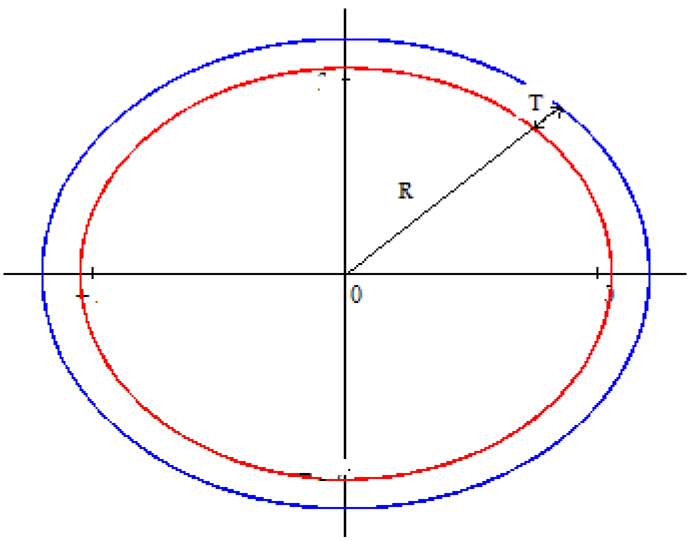
$$t = 1$$

When r1 = 7 and r2 =14 then $\frac{dA}{dt}$ = 0. The significance is before this the Area between the 2 is increasing but after this point the area is decreasing.

4. We can use this same setup to solve a 3-d problem.

Suppose you are blowing a spherical bubble. When the radius is 10 cm it is expanding at 1cm/sec.

At this same time the thickness is 0.1 cm. At what rate is the thickness decreasing at this time?



Here we think of this as the cross-section of our spherical bubble. Let V be the volume of gum

and note $\frac{dV}{dt} = 0.$

$$V = \frac{4}{3} \cdot \pi \cdot (R + T)^3 - \frac{4}{3} \cdot \pi \cdot R^3$$

$$\frac{dV}{dt} = 0 = 4 \cdot \pi \cdot (R + T)^2 \cdot \left(\frac{dR}{dT} + \frac{dT}{dt}\right) - 4 \cdot \pi \cdot (R)^2 \cdot \frac{dR}{dt}$$

$$(R + T)^2 \cdot \left(\frac{dR}{dT} + \frac{dT}{dt}\right) = (R)^2 \cdot \frac{dR}{dt}$$

$$10.1^2 \cdot \left(1 + \frac{dT}{dt}\right) = 10^2 \cdot 1$$

$$\frac{dT}{dt} = \frac{10^2}{10.1^2} - 1 = -.02\frac{cm}{sec}$$

The thickness is decreasing at a rate of .02cm/sec.

We could also use this setup in problems where we consider ice melting on a spherical metallic ball.