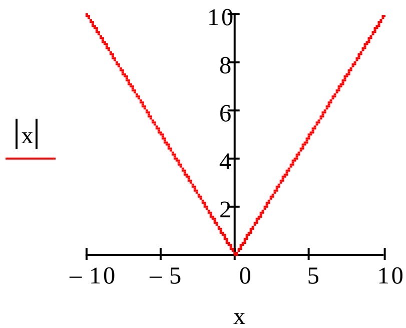
A Point of Non-Differentiability

Consider f(x) = |x| .



Let's Calculate the derivative at x = 0.

by definition: f ' (0) = $\lim_{x \to 0} \frac{(f(x) - f(0))}{x - 0}$

Recal a limit and hence the derivative only exists if the right and left hand limits are equal

So we will consider the right and left hand limits.

$$\lim_{x \to 0^+} \frac{(f(x) - f(0))}{x - 0} = \lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$$ since |x| = x if x >0

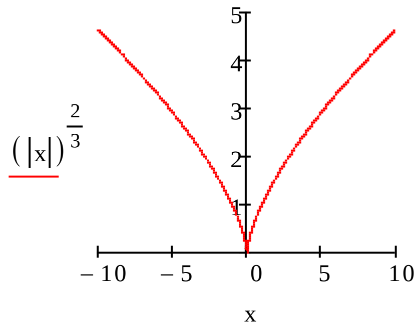
$$\lim_{x \to 0^-} \frac{(f(x) - f(0))}{x - 0} = \lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = -1$$ since |x| = - x if x < 0 .

Therefore $\lim_{x \to 0} \frac{(f(x) - f(0))}{x - 0}$ ***does not exist***.

Therefore f(x) has no derivative at x = 0 and we say f(x) is non-differentiable at x = 0.

If f(x) is continuous at a point but non-differentiable at that point we say f(x) has a cusp at that point.

As another example consider f(x) = $x^{2/3}$ at x = 0.



Do the Calculation or observe the right hand limit is +infinity and the left hand limit is – infinity.
The point is the limit and therefore the derivative does not exist.