A 50 litre tank is initially filled with 10 litres of brine solution containing 20 kg of salt. Starting from time t=0, distilled water is poured into the tank at a constant rate of 4 litres per minute. At the same time, the mixture leaves the tank at a constant rate of k^(1/2) litre per minute,. The time taken for overflow to occur is 20 minutes.

Find Q(t) the amount of salt in the tank 0 < t < 20

Q(t) is the amount of salt so

$$\frac{dQ}{dt} = \text{saltin} - \text{saltout}$$

Since distilled water is being added salt in = 0

Salt out = $\sqrt{k} \cdot \frac{\text{liters}}{\text{min}} \cdot \frac{Q(t)}{V(t)} \frac{\text{lbs}}{\text{litres}}$ Where V(t) is the volume

$$\frac{dQ}{dt} = -\sqrt{k} \cdot \frac{Q}{V}$$

$$Q(0) = 2($$

So now we have to solve an initial value problem for V(t)

$$\frac{dV}{dt} = 4 - \sqrt{k}$$

$$V(0) = 1($$

$$V(t) = \left(4 - \sqrt{k}\right) \cdot t + C$$

$$V(0) = 10 = C$$

$$V(t) = \left(4 - \sqrt{k}\right) \cdot t + 1($$

$$\frac{dQ}{dt} = -\sqrt{k} \cdot \frac{Q}{\left(4 - \sqrt{k}\right) \cdot t + 10}$$

Returning to $V(t) = \left(4 - \sqrt{k}\right)\cdot t + 10$

$$V(20) = 50$$

$$\left(4 - \sqrt{k}\right)\cdot 20 + 10 = 50$$

$$\left(4 - \sqrt{k}\right) = 2$$

$$\sqrt{k} = 2$$

$$k = 4$$

We have:

$$\frac{dQ}{dt} = -\sqrt{k}\cdot\frac{Q}{\left(4 - \sqrt{k}\right)\cdot t + 10} = \frac{-2\cdot Q}{2\cdot t + 10} = \frac{-Q}{t + 5}$$

$$Q(0) = 20$$

Separating

$$\frac{dQ}{Q} = \frac{-dt}{(2\cdot t + 10)} = \left(\frac{-dt}{t + 5}\right)$$

$$\ln(Q) = -\ln(t + 5) + C = \ln\left(\frac{1}{t + 5}\right) + C$$

Note Q and t+5 are positives so we dropped the absolute values

$$Q(t) = \frac{C}{t + 5}$$

Where C = e^C but e^C is a constant so we just use C

$Q(0) = 20 = \frac{C}{5}$

$C = 100$

$Q(t) := \frac{100}{t + 5}$

$V(t) := 2t + 10$