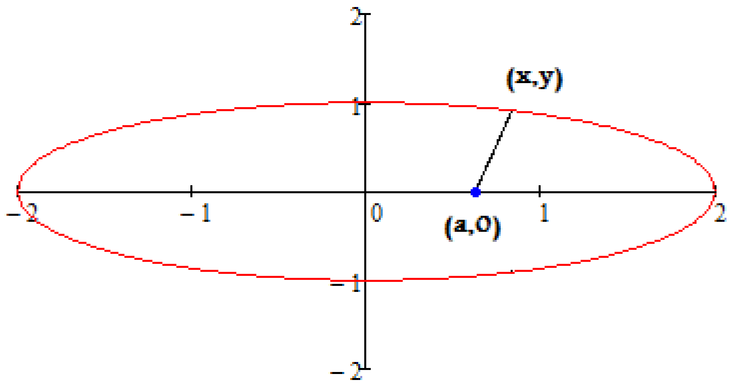
Find the minimum distance from the point (a,0) to the ellipse $\frac{x^2}{4} + y^2 = 1$




Using the square of the distance we have :

$$D = (x-a)^2 + y^2 = (x-a)^2 + 1 - \frac{x^2}{4}$$

$$\frac{dD}{dx} = 2(x-a) - \frac{x}{2}$$

$$2(x-a) - \frac{x}{2} = 0$$

$$x = \frac{4 \cdot a}{3} \qquad y = \frac{\sqrt{9 - 4 \cdot a^2}}{3}, -\frac{\sqrt{9 - 4 \cdot a^2}}{3}$$

$$D\left(\frac{4 \cdot a}{3}\right) = 1 - \frac{a^2}{3}$$

Note that this is the square of the distance so the minimum distance is $\sqrt{1 - \frac{a^2}{3}}$

The points on the ellipse closest to (a,0) is $\left(\frac{4 \cdot a}{3}, \frac{\sqrt{9 - 4 \cdot a^2}}{3}\right)$ and $\left(\frac{4 \cdot a}{3}, \frac{-\sqrt{9 - 4 \cdot a^2}}{3}\right)$

This is not the complete story. Note when $a = 3/2$ $x = 2$ and $y = 0$ so you are at the vertex.

For $a > 3/2$ the x and y coordinates $\left(\frac{4\cdot a}{3}, \frac{\sqrt{9-4\cdot a^2}}{3}\right)$ and $\left(\frac{4\cdot a}{3}, \frac{-\sqrt{9-4\cdot a^2}}{3}\right)$ no longer satisfy the equation of the ellipse.

So for $a > 3/2$ the minimum distance is the distance from (a,0) to (2,0) or | 2-a |.

The minimum distance is:

$$d = \begin{cases} \sqrt{1-\frac{a^2}{3}} & \text{if } a < \frac{3}{2} \\ |2-a| & \text{if } a \ge \frac{3}{2} \end{cases}$$

See the animation Minimum distance from a point to an ellipse