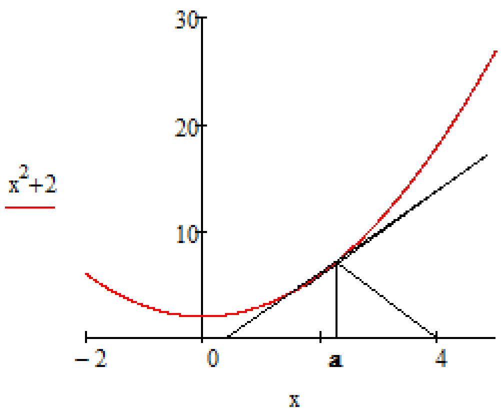
Find the minimum area of the triangle formed by the tangent line to

$f(x) = x^2 + 2$ at (a,f(a)), the x axis and the line segment from the point (a,f(a))

to the point (4,0) For $0 < a < 4$

See the Animation Triangle




First we need the tangent line :

$$y = 2 \cdot a \cdot (x - a) + a^2 + 2$$

Set y = 0 to obtain the x intercept

$$2 \cdot a \cdot (x - a) + \left(a^2 + 2\right) = 0$$

$$\frac{a^2 - 2}{2 \cdot a}$$ x intercept

The base of the triangle is then 4 - xintercept

$$4 - \frac{a^2 - 2}{2 \cdot a}$$ Base

The height of the triangle is f(a) so we have for the Area:

$$\frac{1}{2} \cdot \left(4 - \frac{a^2 - 2}{2 \cdot a}\right) \cdot \left(a^2 + 2\right)$$ Area

$$\left(\frac{a^2-2}{4\cdot a^2}-\frac{1}{2}\right)\cdot\left(a^2+2\right)-2\cdot a\cdot\left(\frac{a^2-2}{4\cdot a}-2\right)$$ Derivative of Area

$$4\cdot a-\frac{1}{a^2}-\frac{3\cdot a^2}{4}$$ Simplified

Now we use the root function on the derivative to find the zero of the derivative and hence the value of a to give us the minimum area: (we define g(a) to be the derivative:

$$g(a) := 4\cdot a-\frac{1}{a^2}-\frac{3\cdot a^2}{4}$$

$a := 1$ Initial guess value

$s := \text{root}(g(a), a)$

$s = 0.658$

To check we also use the Minimize function on the area:

$$A(a) := \frac{1}{2}\cdot\left(4-\frac{a^2-2}{2\cdot a}\right)\cdot\left(a^2+2\right)$$

Using Minimize function on Area

$a := 1$

Given

$0 < a < 4$

$Q := \text{Minimize}(A, a)$

$Q = 0.658$