Find the equation of a line which intersects and is perpindicular to the line

L1 : $x = 7 + t$ $y = -13 + 2 \cdot t$ $z = 8 - 2 \cdot t$

and contains the point (-5,4,2)

Let $a \cdot \vec{i} + b \cdot \vec{j} + c \cdot \vec{k}$ be the vector from (-5,-4,2) to L1

Then $a + 2 \cdot b - 2 \cdot c = 0$

Since the lines intersect but not necessarily at the same time there are time s and t such that:.

$-5 + a \cdot s = 7 + t$

$-4 + b \cdot s = -13 + 2 \cdot t$

$2 + c \cdot s = 8 - 2 \cdot t$

$a + 2 \cdot b - 2 \cdot c = 0$

Since we don't care when we reach the point of intersection take s =1

$-5 + a = 7 + t$

$-4 + b = -13 + 2 \cdot t$

$2 + c = 8 - 2 \cdot t$

$a + 2 \cdot b - 2 \cdot c = 0$

The Strategy is then simply solving the first equation for t and substituting the result in the second equation. Solve the second equation for b in terms of a.

Repeat this with the first and 3d to get c in terms of a.

substitute these results into $a + 2 \cdot b - 2 \cdot c = 0$ and you obtain a = 14 b= -5 and c =2

$x = -5 + 14 \cdot t$

$y = -4 - 5 \cdot t$

$z = 2 + 2 \cdot t$

The particles reaches the point of intersection at t = 1 at the point (9,-9,4)

An intersting variation on this problem would to insist particles with these trajrctories collide at the point of intersection i.e they reach the intersection point at the same time. In this case s = t and we have:

$-5 + a \cdot t = 7 + t$

$-4 + b \cdot t = -13 + 2 \cdot t$

$2 + c \cdot t = 8 - 2 \cdot t$

$a + 2 \cdot b - 2 \cdot c = 0$

See animation : Collision