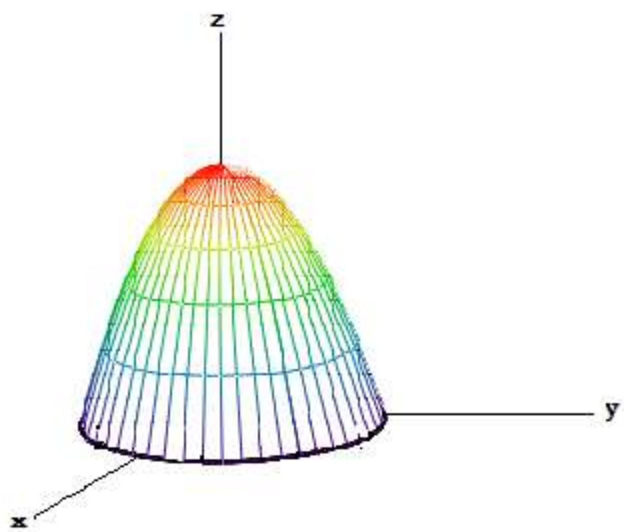
A fluid with density ρ flows with velocity **V** = y **i** + **j** + z **k** . Find the rate of flow of mass upward though the parabaloid $z = 9 - 1/4(x^2 + y^2)$ above the x -y plane.

The density, ρ , is mass/volume and the Flux, Φ , is volume/time therefore the mass/time = ρ Φ

The region of integration in the x-y plane is the circle $x^2 + y^2 = 36$ in the x-y plane since if

$z = 9 - \frac{1}{4}\left(x^2 + y^2\right) = 0$ then $x^2 + y^2 = 36$



$$\vec{V} = y \cdot \vec{i} + \vec{j} + z \cdot \vec{k}$$

Since we are interested in the mass flowing up through the surface we'll orient the surface with an upward normal.

$$\vec{N} = \frac{x}{2} \cdot \vec{i} + \frac{y}{2} \cdot \vec{j} + \vec{k}$$

$$\vec{V} \times \vec{N} = \frac{x \cdot y}{2} + \frac{y}{2} + z = \frac{x \cdot y}{2} + \frac{y}{2} + 9 - \frac{1}{4}\left(x^2 + y^2\right)$$

Since we are integrating over a circle of radius 6 in the x-y plane we convert to polar coordinates

$$\vec{V}\times\vec{N} = r^2\cdot\cos(\theta)\cdot\sin(\theta) + r\cdot\sin(\theta) + 9 - \frac{1}{4}\cdot r^2$$

$$\rho\cdot\Phi = \rho\cdot\int_0^{2\cdot\pi}\int_0^6 \left(r^2\cdot\cos(\theta)\cdot\sin(\theta) + r\cdot\sin(\theta) + 9 - \frac{1}{4}\cdot r^2\right)\cdot r\, dr\, d\theta$$

$$\rho\cdot\int_0^{2\cdot\pi}\int_0^6 \left(r^3\cdot\cos(\theta)\cdot\sin(\theta) + r^2\cdot\sin(\theta) + 9\cdot r - \frac{1}{4}\cdot r^3\right) dr\, d\theta = 162\cdot\pi\cdot\rho$$