Landing a probe on Mars

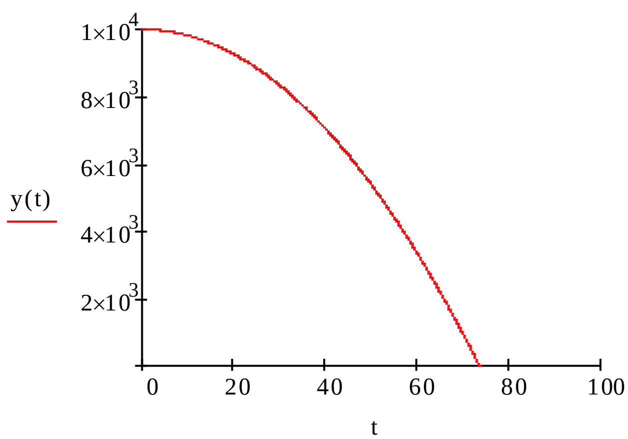
Suppose we want to land a 200kg mass on the surface of Mars. We release it at a height of 10 km.

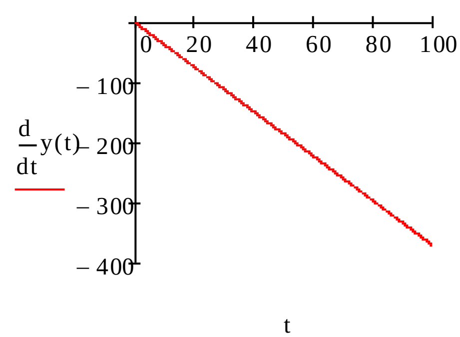
If we simply allow free fall the differential equation and corresponding IVP governing this is $\frac{d^2y}{dt^2} = -3.7 \qquad y(0) = 10000$

We are also assuming the atmosphere of Mars is such that air resistance is negligible.

By simply integrating twice we obtain $y(t) = -3.7 \cdot t^2 + 10000$ and the probe hits the ground in approximately 52 seconds.

$$y(t) := \frac{-3.7}{2} \cdot t^2 + 10000$$





The following tables give us the position and velocity as the probe strikes the ground.

$y(t) =$	$t =$	$\frac{d}{dt}y(t) =$ (m/sec)	$\left(\frac{d}{dt}y(t)\right) \cdot 3.6.6$ (mph)
3.118	73.51	-271.987	-587.492
1.758	73.515	-272.006	-587.532
0.398	73.52	-272.024	-587.572
-0.962	73.525	-272.043	-587.612
-2.323	73.53	-272.061	-587.652

Note the probe hits the ground at 587 mph--not really what we want.

So suppose we fire a series of retro rockets. We start at t= 10 and fire successive bursts every 10 seconds

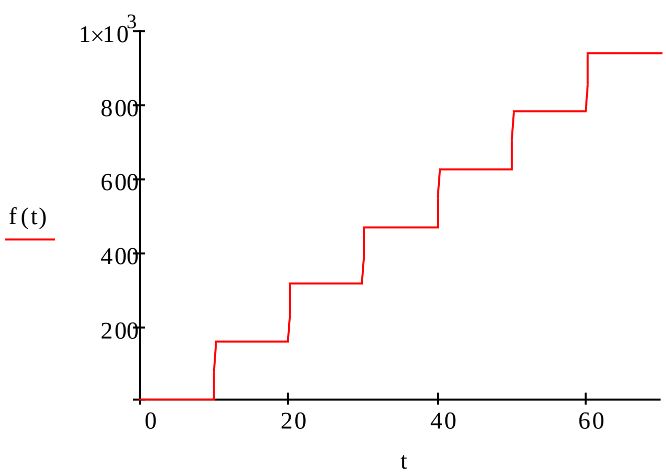
until t = 60. The thrust of each provides a force of 156.532N (a little more than 70lbs of thrust). I'll explain where this value of c

comes from a little later.

$t := 0, .1 .. 80$ $c := 156.53$ and recall in Mathcad the unit step function $u(t) := \Phi(t)$

$$f(t) := c \cdot (\Phi(t-10) + \Phi(t-20) + \Phi(t-30) + \Phi(t-40) + \Phi(t-50) + \Phi(t-60))$$

$$f(t) := c \cdot \sum_{k=1}^{6} u(t - 10 \cdot k)$$



The IVP now takes the form $\frac{d^2y}{dt^2} = -3.7 + \frac{f(t)}{200}$ $y(0) = 1000($ $c := 156.53$

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$$\frac{d^2y}{dt^2} = -3.7 + 0.783 \sum_{k=1}^{6} u(t - 10 \cdot k)$$

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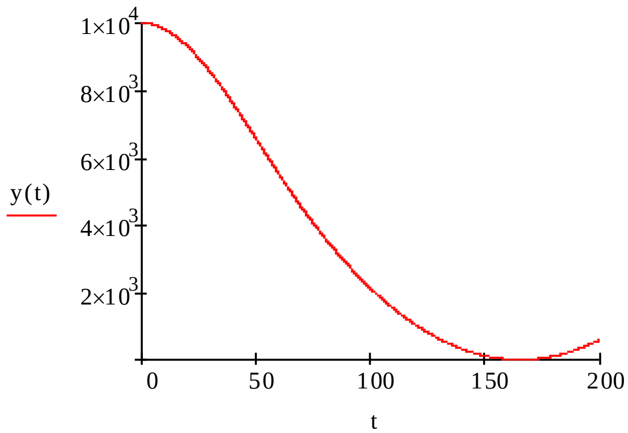
Taking Laplace Transforms we obtain

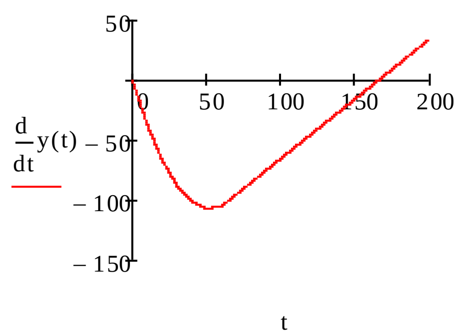
$$s^2 \cdot F = \frac{-3.7}{s} + .783 \left(\sum_{k=1}^{6} \frac{e^{-10 \cdot ks}}{s} \right)$$

$$F = \frac{-3.7}{s^3} + .783 \left(\sum_{k=1}^{6} \frac{e^{-10 \cdot k \cdot s}}{s^3} \right)$$

$L^{-1}\{\frac{1}{s^3}\} = \frac{t^2}{2}$ and it follows $L^{-1}\{\frac{e^{-10 \cdot k \cdot s}}{s^3}\} = \frac{u(t - 10 \cdot k) \cdot (t - 10 \cdot k)^2}{2}$

From which we obtain : $y(t) := 10000 - \frac{3.7 \cdot t^2}{2} + .39133 \sum_{k=1}^{6} \left[\Phi(t - 10 \cdot k) \cdot (t - 10 \cdot k)^2 \right]$





m/sec mph

$y(t) =$

1.482
1.098
0.755
0.451
0.187
-0.037

$t =$

163
163.2
163.4
163.6
163.8
164

$\frac{d}{dt}y(t) =$

-2.017
-1.818
-1.619
-1.42
-1.22
-1.021

$\left(\frac{d}{dt}y(t)\right)\cdot 3.6.6$

-4.357
-3.927
-3.496
-3.066
-2.636
-2.206

Note with this scheme the probe lands at about 163 secs but strikes the surface at just 2.6mph!!

See the Animation : Landing on Mars

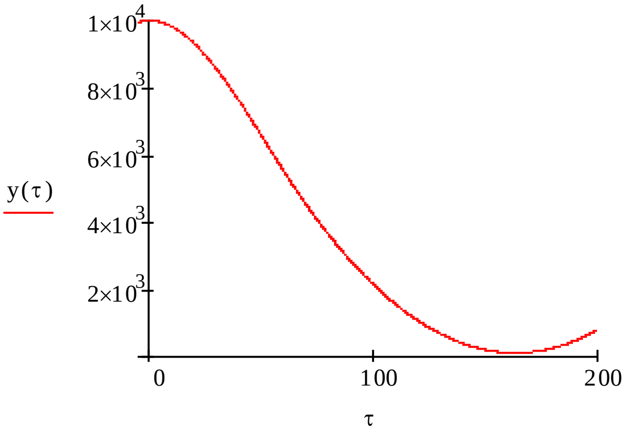
So why should c be 156.53 ? the next 2 graphics illustrate what would happen if c were slightly smaller or slightly larger.

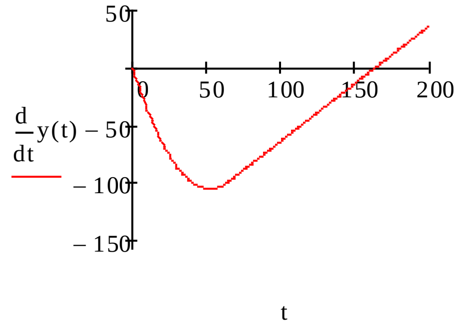
In the first case we take c=157 and note we don't hit the ground.

$c := 157$

$$y(t) := 10000 - \frac{3.7 \cdot t^2}{2} + \frac{c}{400} \cdot \sum_{k=1}^{6} \left[\Phi(t - 10 \cdot k) \cdot (t - 10 \cdot k)^2\right]$$

$$v(t) := \frac{d}{dt}y(t)$$





In the second we hit the ground at about t = 150.235 $y(150.235) = 203.641$ with $v(150.235) = -13.113$ which is about 13 times faster than with our chosen value of c.