Level Surfaces
Suppose w = f(x,y,z) . The problem with graphical analysis is that we need 4 dimensions to graph f. We'll consider 2 ways of trying to obtain a graphical perspective of the behavior of f:

1. Level Surfaces

2. If one of the Arguments is time we can animate i.e. w = f(x,y,t)

Level Surfaces

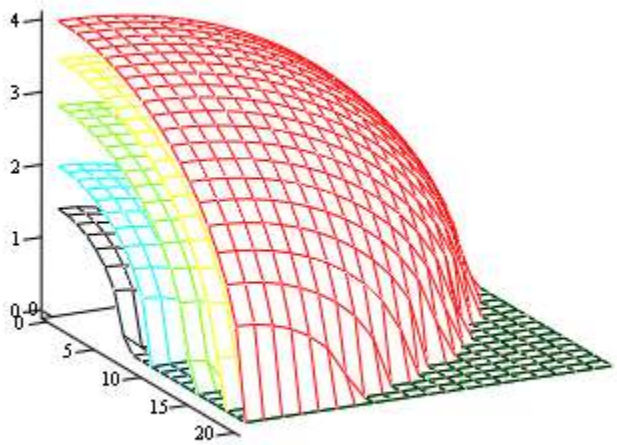
Given w = f(x,y,z) then a level surface is obtained by considering w = c = f(x,y,z). The interpretation being that on a level surface f has the same value at every pt.

For example f could represent the temperature at each pt in 3-space. Then on a level surface the temperature is the same at every pt on that surface

As another example f could be the magnitude of a gravitational or electrostatic field in which case we refer to the level surfaces as equipotential surfaces.

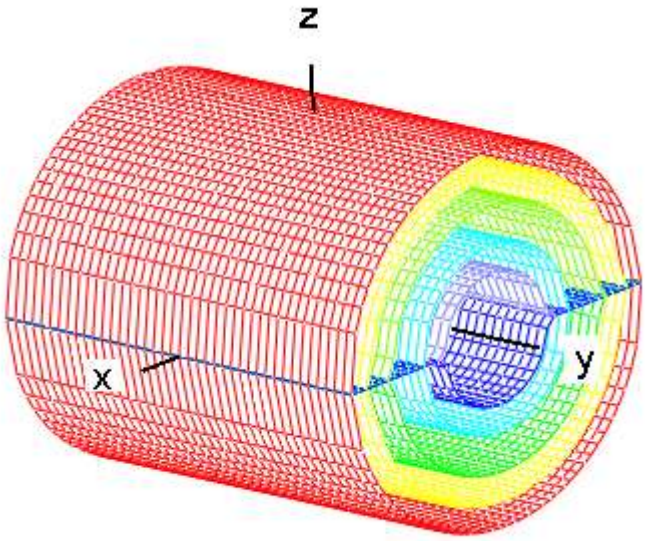
Example 1
$f(x,y,z) = x^2 + y^2 + z^2$. Here the Level surfaces are concentric spheres centered at the origin.

As we move away from the origin the temperature increases and the spheres become closer. The temperature increases at an increasing rate. Here the surfaces corresponds to f = 4,8,12,and 16



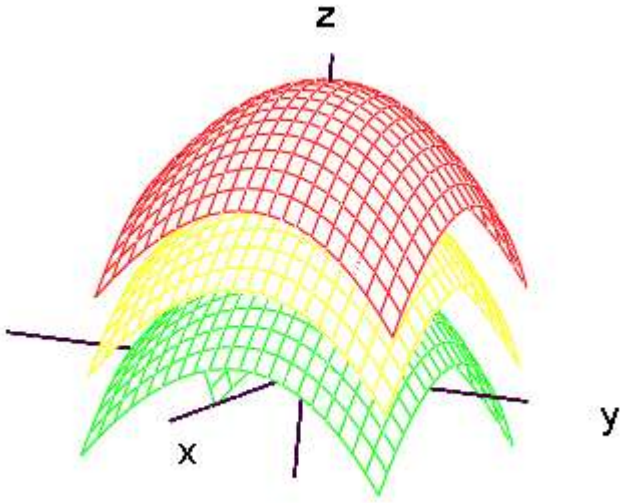

M,N,P,Q,R

Example 2
$f(x,y,z) = x^2 + z^2$ , the level Surfaces are the concentric cylinders $x^2 + z^2 = c$ with the main axis along the y- axis. With some adjustments of constants these level surfaces could represent the electric field of a line of charge along the y axis. Here we have f = 2,4,8,12, and 16.




Example 3

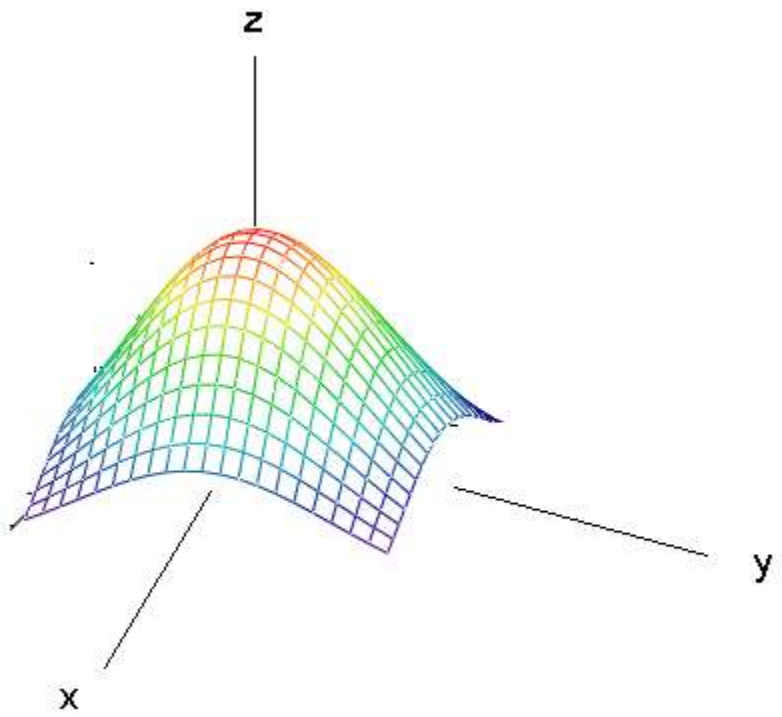
f(x.y,z) = $x^2$ +$y^2$ +z . The level surfaces are the parabaloids z = c - $x^2$ -$y^2$ .




Example 4

Suppose we have f(x,y,t) = $\cos(t)\cdot e^{-x^2-y^2}$ which represents the temperature at any pt on a rectangular plate in the plane. At each fixed $t_0$ we have a function of 2 variables

f(x,y,$t_0$) = cos($t_0$) $e^{-x^2-y^2}$. For example below is the temperature profile at t =0. However we can let t vary . See Animation 3-D Normal with corresponding contour diagram




Example 5

Just for fun See Animation labeled Flying. What is a possible formula for f(x,y,t) ?