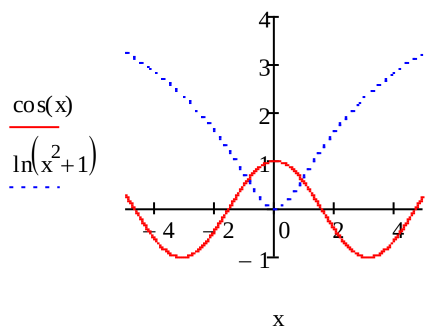
Let R be the region bounded by the curves f(x) = ln($x^2$+1) and g(x) = cos(x)

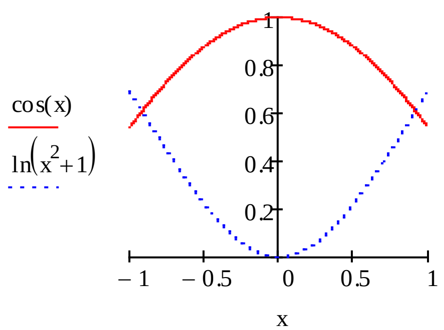
a. Find the Area of R

B Suppose R is the base of a solid and the cross-sections taken perpendicular to the base are isosceles right triangles-- Set up but do not evaluate the integral which calculates the Volume.

a.




1. Consider the graph




Use your favorite graphical or numerical method to solve for points of intersection

In mathcad we have the root function which yields $\pm$.916

we have

$$A = \int_{-.916}^{.916} \cos(x) - \ln\left(x^2 + 1\right) dx = \int_{-.916}^{.916} \cos(x)\, dx - \int_{-.916}^{.916} \ln\left(x^2 + 1\right) dx$$

$$\int_{-.916}^{.916} \cos(x)\, dx = \sin(x) \Big|_{-.916}^{.916} = 2\sin(.916) = 1.586$$

For $\int_{-.916}^{.916} \ln\left(x^2 + 1\right) dx$ we'll integrate by parts to find the anti-derivative first

$u = \ln\left(x^2 + 1\right)$ $\qquad$ $dv = dx$

$du = \dfrac{2 \cdot x}{x^2 + 1}$ $\qquad$ $v = x$

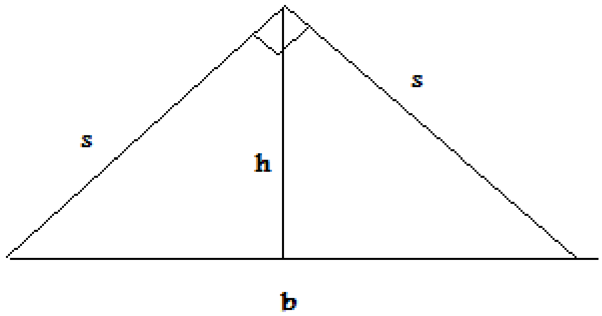
$$\int \ln\left(x^2 + 1\right) dx = x \cdot \ln\left(x^2 + 1\right) - \int \frac{2 \cdot x^2}{1 + x^2} dx = x \cdot \ln\left(x^2 + 1\right) - \int 2 - \frac{2}{1 + x^2} dx$$

$$\int \ln\left(x^2 + 1\right) dx = x \cdot \ln\left(x^2 + 1\right) - 2 \cdot x - 2\arctan(x)$$

$$\int_{-.916}^{.916} \ln\left(x^2 + 1\right) dx = \left(x \cdot \ln\left(x^2 + 1\right) - 2 \cdot x - 2\arctan(x)\right) \Big|_{-.916}^{.916} = .418$$

$$\int_{-.916}^{.916} \cos(x) - \ln\left(x^2 + 1\right) dx = 1.586 - .418 = 1.168$$

For the second part




In General $A = \frac{1}{2} \cdot b \cdot h$

For an isosceles right triangle: l

$h = \frac{b}{2} \cdot \tan(45) = \frac{b}{2}$

$A = \frac{1}{2} \cdot b \cdot h = \frac{1}{4} \cdot b^2$

Here $b = \cos(x) - \ln(x^2 + 1)$

$A(x) = \frac{1}{4} \cdot \left[ \left( \cos(x) - \ln(x^2 + 1) \right)^2 \right]$

$V = \frac{1}{4} \cdot \int_{-.916}^{.916} \left[ \left( \cos(x) - \ln(x^2 + 1) \right)^2 \right] dx$