

Animation for Demonstration of Rigorous Limits for Sequences

We want to demonstrate the rigorous definition of limit for sequences. We will use as our example

$$\lim_{n \rightarrow \infty} \left(1 - \frac{\sin(n)}{n} \right) = 1$$

We start by defining ε : $\varepsilon := .1 - \frac{\text{FRAME}}{1000}$ This allows ε to decrease from .1 in increments of .001

We then want M in terms of ε we use $M := \frac{2}{\varepsilon}$.

The reason we use $M := \frac{1}{\varepsilon}$ is $|L - a_n| = \left| 1 - \left(1 - \frac{\sin(n)}{n} \right) \right| = \left| \frac{\sin(n)}{n} \right| < \frac{1}{n}$ Therefore if $M > 1/\varepsilon$

Then $\frac{1}{n} < \varepsilon$ if $n > M$ We use $M := \frac{2}{\varepsilon}$ because M has to be greater than $1/\varepsilon$ --(We always use overkill)

Now we define the number of points to be graphed

$$n := 0..100$$

We define the sequence

$$f(n) := 1 - \frac{\sin(n)}{n}$$

Now we set the graph

On the vertical we put

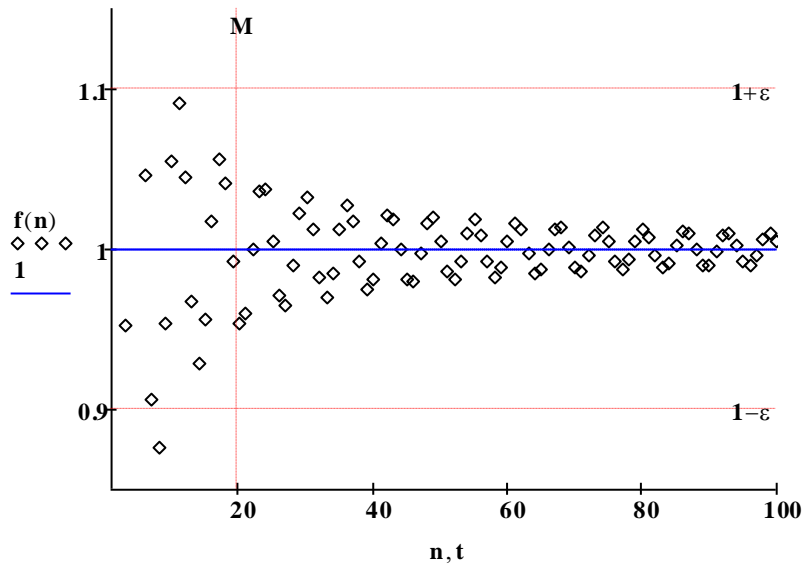
1. $f(n)$ in one place holder and 1 on the other. for Plot 1 we change to points and add a symbol.
2. In the show marker place holders we put $1 + \varepsilon$ and $1 - \varepsilon$
3. For the Max and Min y values we put 1.15 and .85 Since the initial value of ε is .1

For the Horizontal on the show markers we put M.

Initially the graph appears as below:

$$\varepsilon = 0.1$$

$$M = 20$$



For the animation we would then have as the Format:

$$\varepsilon := .1 - \frac{\text{FRAME}}{1000}$$

$$M := \frac{2}{\varepsilon}$$

$$n := 0..100$$

$$f(n) := 1 - \frac{\sin(n)}{n}$$

We then use 79 frames because at 80 frames $\varepsilon = .1 - \frac{80}{1000} = .02$ and $M = \frac{2}{\varepsilon} = 100$

I would then animate at 6 frames/sec

