

Limits at ∞

Intuitive approach

When we write $\lim_{x \rightarrow \infty} f(x) = L$ intuitively we are saying as x increases without bound $f(x)$ approaches the value L

. We have a similar interpretation as $x \rightarrow -\infty$. In the first 2 animations we see

$$\lim_{x \rightarrow \infty} f(x) = 3 \text{ and } \lim_{x \rightarrow -\infty} f(x) = 3$$

Rigorous approach

By $\lim_{x \rightarrow \infty} f(x) = L$ we mean given any $\varepsilon > 0$ there is a number M such that $L - \varepsilon < f(x) < L + \varepsilon$ whenever $x > M$. Again we usually state this as $|f(x) - L| < \varepsilon$ whenever $x > M$

The 2 animations Limit at ∞ and Limit at $-\infty$ show that $1 - \sin(x)/x$ converges to 1