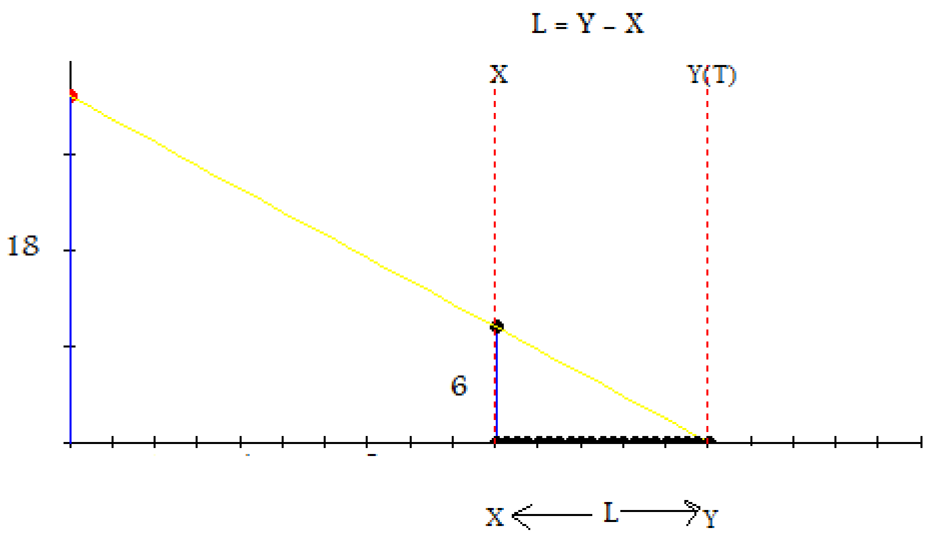
Walking toward and away from a lightpole

Suppose a 6 ft man is 10 ft away from an 18 ft lightpole. We are going to consider 2 problems.

1. See the animation walking toward the light .The man walks toward the light pole at 1ft/sec

a. Find the rate at which the length of his shadow is changing

b. Find the speed at which the tip of his shadow is moving



a. Let X denote the position of the Man, Y denote the tip of the shadow, and L the length of the shadow.

By similar triangles $\frac{18}{X+L} = \frac{6}{L}$.

From which we obtain $L = \frac{X}{2}$.

Differentiating we obtain $\frac{dL}{dt} = \frac{1}{2} \cdot \frac{dX}{dt} = -.5\frac{ft}{sec}$. Note dX/dt = -1 since X is decreasing.

b. $Y = X + L$

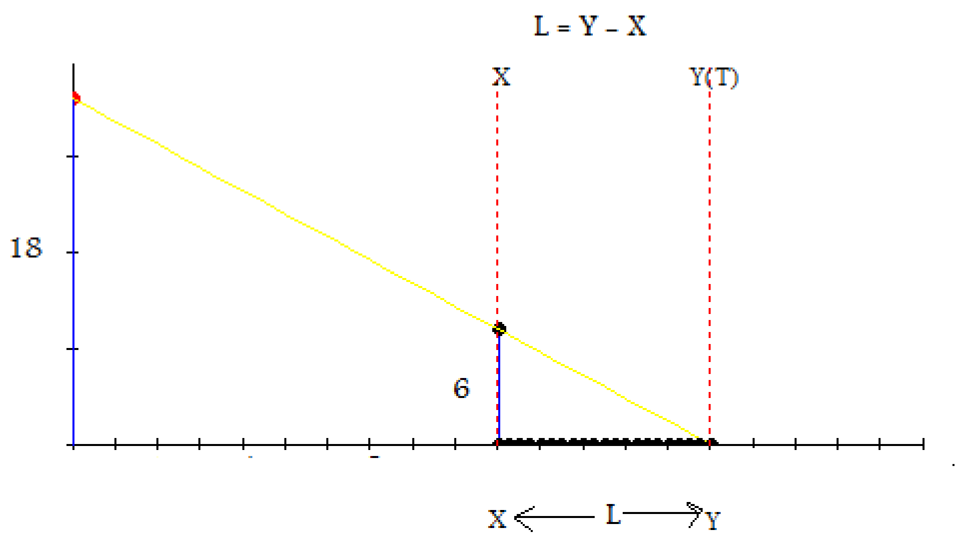
$$\frac{dY}{dt} = \frac{dX}{dt} + \frac{dL}{dt} = -1 - .5 = -1.5\frac{ft}{sec}$$

Note the length of the shadow is decreasing and the tip of the shadow is catching up to the man as he approaches the light.

1. See the animation walking away from the light .The man walks away from the light pole at 1ft/sec

a. Find the rate at which the length of his shadow is changing

b. Find the speed at which the tip of his shadow is moving



a. Again Let X denote the position of the Man, Y denote the tip of the shadow, and L the length of the shadow.

We still have by similar triangles $\frac{18}{X+L} = \frac{6}{L}$.

From which we obtain $L = \frac{X}{2}$.

Differentiating we obtain $\frac{dL}{dt} = \frac{1}{2} \cdot \frac{dX}{dt} = .5$. Note dX/dt = 1 since X is increasing.

b. $Y = X + I$

$$\frac{dY}{dt} = \frac{dX}{dt} + \frac{dL}{dt} = 1 + .5 = 1.5 \frac{\text{ft}}{\text{sec}}$$

Note the length of the shadow is increasing and the tip of the shadow is moving further ahead of the man as he moves away from the light.