Using Laplace Transforms

Consider the IVP y '' + 5y' + 6 y = e $^{-t}$

y(0) = 1 y ' (0) = 1

We start by Transforming by hand after all it is the inverse transform which gives the most trouble. Use F not F(s) as is traditionally used

$$\left(s^2+5\cdot s+6\right)\cdot F-s-1-5=\frac{1}{s+1}$$

Highlight F then use SYMBOLICS-VARIABLE-SOLVE to solve for F yields

$$\frac{\left(s^2+7\cdot s+7\right)}{\left[(s+1)\cdot\left(s^2+5\cdot s+6\right)\right]}$$

Highlight s SYMBOLICS- VARIABLE- INVERSE TRANSFORM
yields

$$\frac{1}{2}\cdot\exp(-t)-\frac{5}{2}\cdot\exp(-3\cdot t)+3\cdot\exp(-2\cdot t)$$

Define $y(t):=\frac{1}{2}\cdot\exp(-t)-\frac{5}{2}\cdot\exp(-3\cdot t)+3\cdot\exp(-2\cdot t)$ DONE!!!

Example 2 A little harder

y '' + 2y' + y = $e^{-2\cdot t}\cdot\cos(3\cdot t)$ y(0) = 2 y ' (0) = 1

$\left(s^2+2\cdot s+1\right)\cdot F-s-1-2=\blacksquare$ here you may not recall L{$e^{-2\cdot t}\cdot\cos(3\cdot t)$}

No Problem copy $e^{-2\cdot t}\cdot\cos(3\cdot t)$ highlight t and use SYMBOLICS- TRANSFORM- LAPLACE

$$e^{-2\cdot t}\cdot\cos(3\cdot t)$$

has Laplace transform

$$\frac{(s+2)}{\left[(s+2)^2+9\right]}$$

Now fill it in

$\left(s^2+2\cdot s+1\right)\cdot F-s-1-2=\frac{(s+2)}{\left[(s+2)^2+9\right]}$ Now proceed as before : solve for F

has solution(s)

$$\frac{\left(s^3 + 7\cdot s^2 + 26\cdot s + 41\right)}{\left[\left(s^2 + 4\cdot s + 13\right)\cdot\left(s^2 + 2\cdot s + 1\right)\right]}$$ Now Find the Inverse Transform

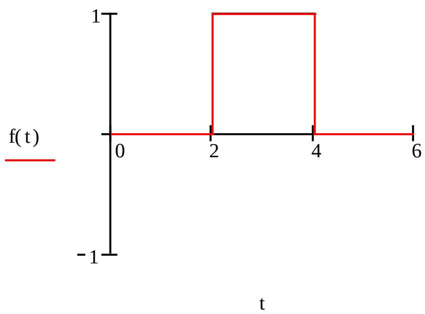
has inverse Laplace transform

$$\frac{21}{10}\cdot t\cdot \exp(-t) + \frac{27}{25}\cdot\exp(-t) - \frac{2}{25}\cdot\exp(-2\cdot t)\cdot\cos(3\cdot t) - \frac{3}{50}\cdot\exp(-2\cdot t)\cdot\sin(3\cdot t)$$

$$y(t) := \frac{21}{10}\cdot t\cdot \exp(-t) + \frac{27}{25}\cdot\exp(-t) - \frac{2}{25}\cdot\exp(-2\cdot t)\cdot\cos(3\cdot t) - \frac{3}{50}\cdot\exp(-2\cdot t)\cdot\sin(3\cdot t)$$

Even with discontinuous functions This works : Here $\Phi(t)$ is the unit step function

$f(t) := \Phi(t-2) - \Phi(t-4)$



Consider $\frac{dy}{dt} + 2\cdot y := f(t)$ y(0) = 3

$(s+2)\cdot F - 3 =$ ▮

$\Phi(t-2) - \Phi(t-4)$

has Laplace transform

$$\frac{\exp(-2\cdot s)}{s} - \frac{\exp(-4\cdot s)}{s}$$

$$(s+2)\cdot F - 3 = \frac{\exp(-2\cdot s)}{s} - \frac{\exp(-4\cdot s)}{s}$$

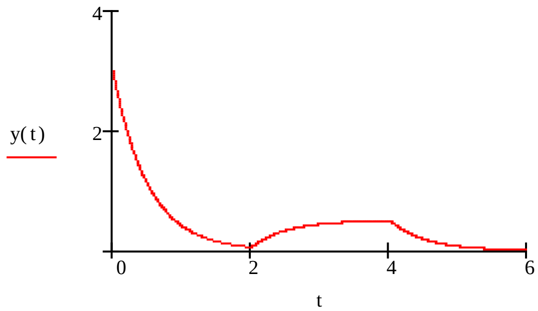
has solution(s)

$$\frac{-(-3\cdot s - \exp(-2\cdot s) + \exp(-4\cdot s))}{(s\cdot(s+2))}$$

has inverse Laplace transform

$$3\cdot\exp(-2\cdot t) + \frac{1}{2}\cdot\Phi(t-2) - \frac{1}{2}\cdot\Phi(t-2)\cdot\exp(-2\cdot t + 4) - \frac{1}{2}\cdot\Phi(t-4) + \frac{1}{2}\cdot\Phi(t-4)\cdot\exp(-2\cdot t + 8)$$

$$y(t) := 3\cdot\exp(-2\cdot t) + \frac{1}{2}\cdot\Phi(t-2) - \frac{1}{2}\cdot\Phi(t-2)\cdot\exp(-2\cdot t + 4) - \frac{1}{2}\cdot\Phi(t-4) + \frac{1}{2}\cdot\Phi(t-4)\cdot\exp(-2\cdot t + 8)$$



Note this could represent an RC circuit where the capacitor discharges then a voltage source is turned on at t = 2 it starts charging then the voltage source is turned off at t =4 and then discharges.