Lab 5 Parametric Equations - curves in 3-space
Before Beginning familiarize yourself with the page on mathematics of parametric equations in 3 space

To graph a curve in 3 space we use the surface plot, but this time x, y, and z are functions of a single variable t. We use the following template:

$s := 0..\blacksquare$ Set up the number of pts to evaluate

$t(s) := \frac{s}{10}$ Sets up the interval at which we evaluate the component functions here every 0.1 of a second.

Note if we wanted to evaluate and plot our functions in intervals of one second we wouldn't need both s and t(s) however, very rarely is this the case as our curve would not be very smooth.

$i := 0$ this defines a "dummy variable” to create a multi-dimensional array

$x_{i,s} := \blacksquare$ $y_{i,s} := \blacksquare$ $z_{i,s} := \blacksquare$ Define the 3 component functions

Example
Graph the helix in three space defined parametrically by

x(t) = cos(t)
y(t) = sin(t)
z(t) = t/10

$s := 0..200$ Here we are using 200 pts

$t(s) := \frac{s}{10}$ Sets up the interval at which we evaluate the component functions here every 0.1 of a second

$i := 0$
$x_{i,s} := \cos(t(s))$ $y_{i,s} := \sin(t(s))$ $z_{i,s} := t(s)$ Note fns expressed in terms of t(s) not s

Now Choose Surface Plot and put (x,y,z) in the place holder --***<u>Parentheses must be used</u>***

(x, y, z)

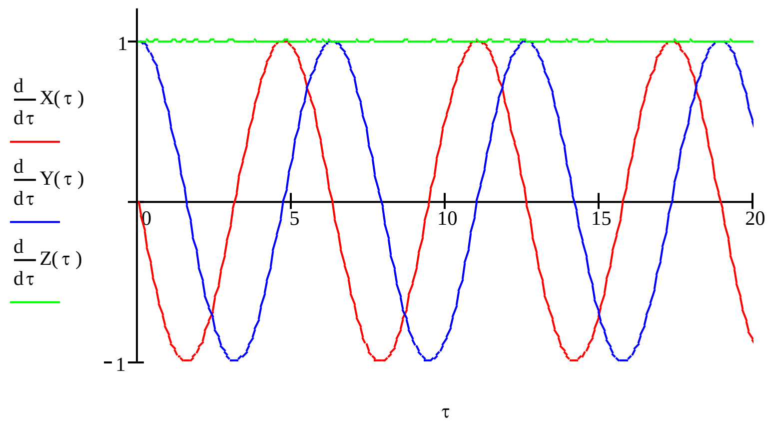
Under the Format window I've clicked SHOW BORDER off and under Axes Style I've clicked on NONE.
Under Appearance I've selected the color map.

But you can format however you feel is appropriate.

Of course you'd want to locate the initial pt (1,0,0) and analyze the derivatives.

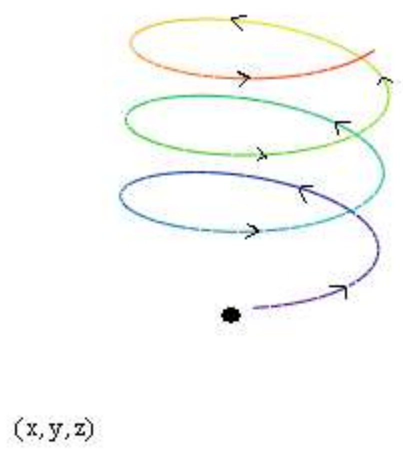
To analyze the derivatives we'll plot all 3 on a 2-D graph and use the variable τ since t is already in use and X,Y, and Z for the component functions since x, y, and z are already in use.

$\tau := 0, .1 .. 20 \quad X(\tau) := \cos(\tau) \quad Y(\tau) := \sin(\tau) \quad Z(\tau) := \tau$



Note the trajectory is smooth as the derivatives are never simultaneously 0.

Initially $\frac{d}{d\tau}X(\tau) < 0$ $\frac{d}{d\tau}Y(\tau) > 0$ and $\frac{d}{d\tau}Z(\tau) > 0$ so the trajectory is counterclockwise increasing at a constant rate in the z direction



Note I copied the graph from Mathcad into Paint Brush to add the initial point and direction arrows and then pasted back into the Mathcad Program.

Example 2
Let's graph the curve defined in vector form by
$\overrightarrow{r(t)} = t \cdot \vec{i} + t^2 \cdot j + \left[(1) - t^2 + t^4\right] \cdot \vec{k}$

(Verify that the motion is on the surface of a saddle)

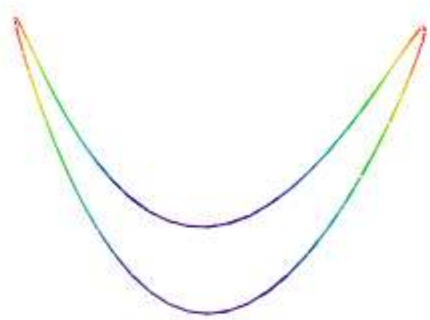
$s := 0..100$ Here we are using 100 pts

$t1(s) := \frac{s}{10}$ Sets up the interval at which we evaluate the component functions here every 0.1 of a second

$i := 0$

$x1_{i,s} := \cos(t(s))$ $y1_{i,s} := \sin(t(s))$ $z1_{i,s} := 1 - \cos(t(s))^2 + \sin(t(s))^2$

Note here I'm using x1,y1,z1, and t1--the reason is we're using less pts than in the previous example and so if we used x,y,z, and t Mathcad would graph this example for the first 100 pts and the previous example for the next 100 creating a hybrid manimal like in "The Island of Dr Moreau" (I know some of you already know this)



$(x1, y1, z1)$

Exercises
1. Graph the curve defined parametrically by :

x = t
y = t
z = sin(t)

I'll give you the Set-up on this one
$s := 0..120$
$t(s) := \frac{s}{20}$
(Note here the increment is 1/20 of a second)

$i := 0..1$

$x_{s,i} := t(s)$
$y_{s,i} := t(s)$
$z_{s,i} := \sin(t(s))$

2. Graph the curve defined parametrically by:

$x(t) = \cos^3(t)$
$y(t) = \sin^3(t)$
$z(t) = t/20$

Use the same set-up as in the example except let s vary from 0 to 240. Make sure to include initial point and direction. Use an increment of 1/20 of a second.

3. Graph
$x(t) = \cos(3t)$
$y(t) = \sin(3t)$
$z(t) = \cos(5t)$

Set S to be initially 200 with an increment of 1/30 of a second.

To understand how the curve is generated in time change s to 50, then increase s by 10 increments and watch how the curve changes until s is 200 again. Use say x3,y3,z3,and t3 To avoid manimals.

To animate Set s := 0..FRAME and use the RECORD ANIMATION in the TOOLS Window.