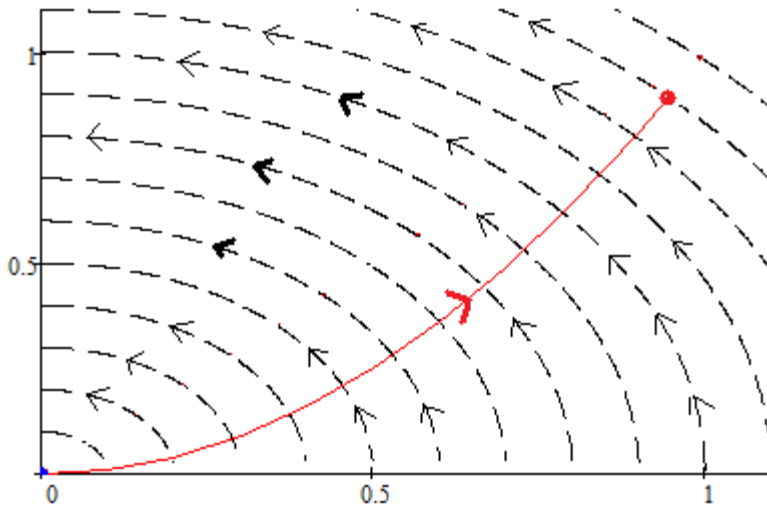


## The Line Integral and Independence of Parameterization

Suppose we have  $\vec{F} = -y\vec{i} + x\vec{j}$  and  $C$  is the parabolic arc  $y = x^2$  from  $(0,0)$  to  $(1,1)$ . Does the way in which we parameterize  $C$  determine the value of the Line Integral?

We consider 3 parameterizations.



$$\vec{i} := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \vec{j} := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Ex 1

$$x(t) := t \quad y(t) := t^2$$

$$\vec{r}(t) := x(t)\vec{i} + y(t)\vec{j} \quad \vec{r}_1(t) := \left(\frac{d}{dt}x(t)\right)\vec{i} + \left(\frac{d}{dt}y(t)\right)\vec{j} \quad \vec{F}_{xx}(t) := -y(t)\vec{i} + x(t)\vec{j}$$

$$\int_0^1 \vec{F}(t) \cdot \vec{r}_1(t) dt = 0.333$$

Ex 2

$$\underline{x}(t) := \sin(t) \quad \underline{y}(t) := \sin(t)^2 \quad 0 \leq t \leq \pi/2$$

$$\underline{r}(t) := \underline{x}(t) \cdot \mathbf{i} + \underline{y}(t) \cdot \mathbf{j} \quad \underline{r}'(t) := \left( \frac{d}{dt} \underline{x}(t) \right) \cdot \mathbf{i} + \left( \frac{d}{dt} \underline{y}(t) \right) \cdot \mathbf{j} \quad \underline{F}(t) := -\underline{y}(t) \cdot \mathbf{i} + \underline{x}(t) \cdot \mathbf{j}$$

$$\int_0^{\pi/2} \underline{F}(t) \cdot \underline{r}'(t) dt = 0.333$$

Ex 3

$$\underline{x}(t) := e^t - 1 \quad \underline{y}(t) := (e^t - 1)^2 \quad 0 \leq t \leq \ln(2) \quad \underline{r}(t) := \underline{x}(t) \cdot \mathbf{i} + \underline{y}(t) \cdot \mathbf{j}$$
$$\underline{r}'(t) := \left( \frac{d}{dt} \underline{x}(t) \right) \cdot \mathbf{i} + \left( \frac{d}{dt} \underline{y}(t) \right) \cdot \mathbf{j}$$
$$\underline{F}(t) := -\underline{y}(t) \cdot \mathbf{i} + \underline{x}(t) \cdot \mathbf{j}$$

$$\int_0^{\ln(2)} \underline{F}(t) \cdot \underline{r}'(t) dt = 0.333$$

## The line Integral is independent of parameterization

Be Careful we are not saying the line integral is independent of path ( we'll explore this later) what we are saying for a given path the line integral is independent of the parameterization of that path,

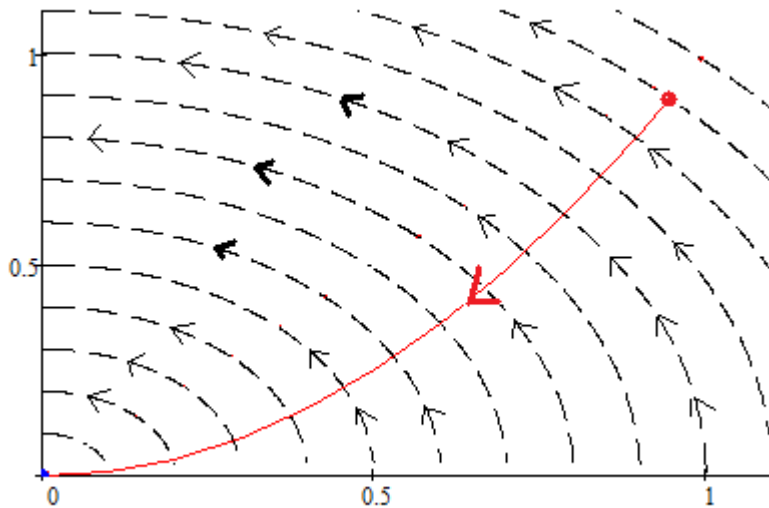
### Reversing the orientation along a curve

In these exercises we show if you reverse the orientation along a curve C then:

$$\int_C \vec{F} \cdot \frac{d\vec{r}}{dt} dt = - \int_{-C} \vec{F} \cdot \frac{d\vec{r}}{dt} dt$$

Compare this with the result for real valued functions  $\int_b^a f(x) dx = - \int_a^b f(x) dx$

The 3 examples correspond to the 3 above respectively



$$\mathbf{i} := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \mathbf{j} := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Ex 1

$$\underline{\underline{x}}(t) := 1 - t \quad \underline{\underline{y}}(t) := (1 - t)^2 \quad \underline{\underline{r}}(t) := x(t) \cdot \mathbf{i} + y(t) \cdot \mathbf{j} \quad \underline{\underline{r}}'(t) := \left( \frac{d}{dt} x(t) \right) \cdot \mathbf{i} + \left( \frac{d}{dt} y(t) \right) \cdot \mathbf{j}$$

$$\underline{\underline{F}}(t) := -y(t) \cdot \mathbf{i} + x(t) \cdot \mathbf{j}$$

$$\int_0^1 \underline{\underline{F}}(t) \cdot \underline{\underline{r}}'(t) dt = -0.333$$

Ex 2

$$\underline{\underline{x}}(t) := 1 - \sin(t) \quad \underline{\underline{y}}(t) := (1 - \sin(t))^2 \quad \underline{\underline{r}}(t) := \underline{\underline{x}}(t) \cdot \mathbf{i} + \underline{\underline{y}}(t) \cdot \mathbf{j}$$

$$\underline{\underline{r}}'(t) := \left( \frac{d}{dt} \underline{\underline{x}}(t) \right) \cdot \mathbf{i} + \left( \frac{d}{dt} \underline{\underline{y}}(t) \right) \cdot \mathbf{j} \quad \underline{\underline{F}}(t) := -\underline{\underline{y}}(t) \cdot \mathbf{i} + \underline{\underline{x}}(t) \cdot \mathbf{j}$$

$$\int_0^{\frac{\pi}{2}} \underline{\underline{F}}(t) \cdot \underline{\underline{r}}'(t) dt = -0.333$$

Ex 3

$$\underline{\underline{x}}(t) := 2 - e^t \quad \underline{\underline{y}}(t) := (2 - e^t)^2 \quad \underline{\underline{r}}(t) := \underline{\underline{x}}(t) \cdot \mathbf{i} + \underline{\underline{y}}(t) \cdot \mathbf{j} \quad \underline{\underline{r}}'(t) := \left( \frac{d}{dt} \underline{\underline{x}}(t) \right) \cdot \mathbf{i} + \left( \frac{d}{dt} \underline{\underline{y}}(t) \right) \cdot \mathbf{j}$$

$$\underline{\underline{F}}(t) := -\underline{\underline{y}}(t) \cdot \mathbf{i} + \underline{\underline{x}}(t) \cdot \mathbf{j}$$

$$\int_0^{\ln(2)} \underline{\underline{F}}(t) \cdot \underline{\underline{r}}'(t) dt \rightarrow -\frac{1}{3}$$