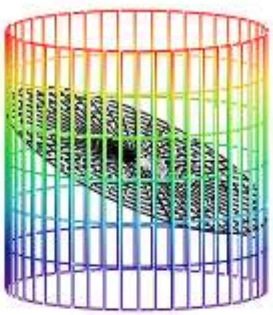
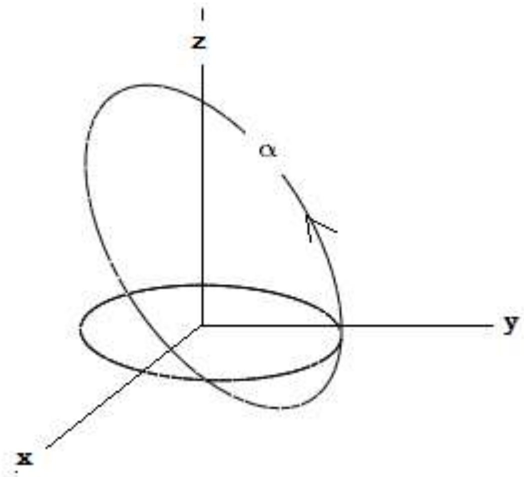
Let the curve α be the intersection of the cylinder $x^2 + y^2 = 1$ and the plane z = 1 - x - y.

Let $\vec{F} = (x - y)\cdot\vec{i} + (y - z)\cdot\vec{j} + (z - x)\cdot\vec{k}$ Calculate the circulation of $\vec{F}$ around α .

1. Compute the line Integral directly

2.Use Stokes Theorem





1. Direct Computation

$$x = \cos(t)$$

$$y = \sin(t)$$

$$z = 1 - \cos(t) - \sin(t)$$

$$\vec{r} = \cos(t)\cdot\vec{i} + \sin(t)\cdot\vec{j} + (1 - \cos(t) - \sin(t))\cdot\vec{k}$$

$$\vec{F} = (x - y)\cdot\vec{i} + (y - z)\cdot\vec{j} + (z - x)\cdot\vec{k}$$

$$\frac{d\vec{r}}{dt} = -\sin(t)\cdot\vec{i} + \cos(t)\cdot\vec{j} + (\sin(t) - \cos(t))\cdot\vec{k}$$

$$\vec{F} = (\cos(t) - \sin(t))\cdot\vec{i} + (2\cdot\sin(t) - 1 + \cos(t))\cdot\vec{j} + (1 - 2\cdot\cos(t) - \sin(t))\cdot\vec{k}$$

$$\vec{F}\cdot\frac{d\vec{r}}{dt} = \sin^2(t) - \sin(t)\cdot\cos(t) + 2\cdot\sin(t)\cdot\cos(t) - \cos(t) + \cos^2(t) + \sin(t) - 2\cdot\cos(t)\sin(t) - \sin^2(t) - \cos(t) + 2\cos^2(t) + \cos(t)\cdot\sin(t)$$

$$\vec{F}\cdot\frac{d\vec{r}}{dt} = 3\cdot\cos^2(t) - 2\cdot\cos(t) + \sin(t)$$

$$\int_C \vec{F}\cdot\frac{d\vec{r}}{dt}\,dt = \int_0^{2\cdot\pi} 3\cos^2(t) - 2\cdot\cos(t) + \sin(t)\,dt = 3\cdot\pi$$

2. Using Stokes Thm

$$\nabla \times \vec{F} = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ x - y & y - z & z - x \end{pmatrix} = \vec{i} + \vec{j} + \vec{k}$$

Since z = 1 - x - y $\quad \vec{N} = \vec{i} + \vec{j} + \vec{k}$

$$\left(\nabla \times \vec{F}\right)\cdot\vec{N} = 3$$

$$\int_{C} \vec{F} \cdot \overrightarrow{\frac{dr}{dt}}\, dt = \int\int_{S} (\nabla \times \vec{F}) \cdot \vec{n}\, dS = \int\int_{R} (\nabla \times \vec{F}) \cdot \vec{N}\, dA = \int_{0}^{2\cdot\pi} \int_{0}^{1} 3\cdot r\, dr\, d\theta = 3\cdot\pi$$