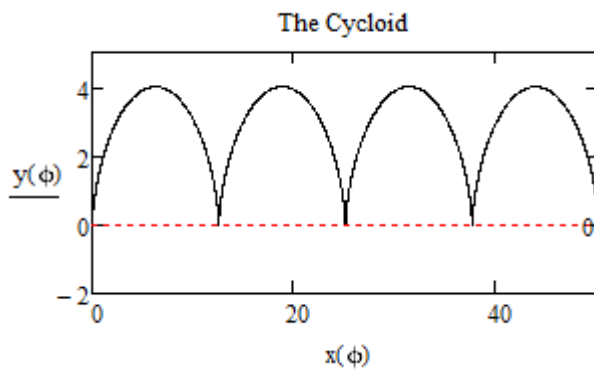


## The Cycloid

We have been parameterizing curves in terms of time. But we can also use things like arclength or angles to parameterize curves.

In this discussion we will use an angle to parameterize the cycloid. A cycloid is the curve traced out by a fixed point on a circle as the circle rolls along the x-axis

[See animation cycloid](#) and the graph below:



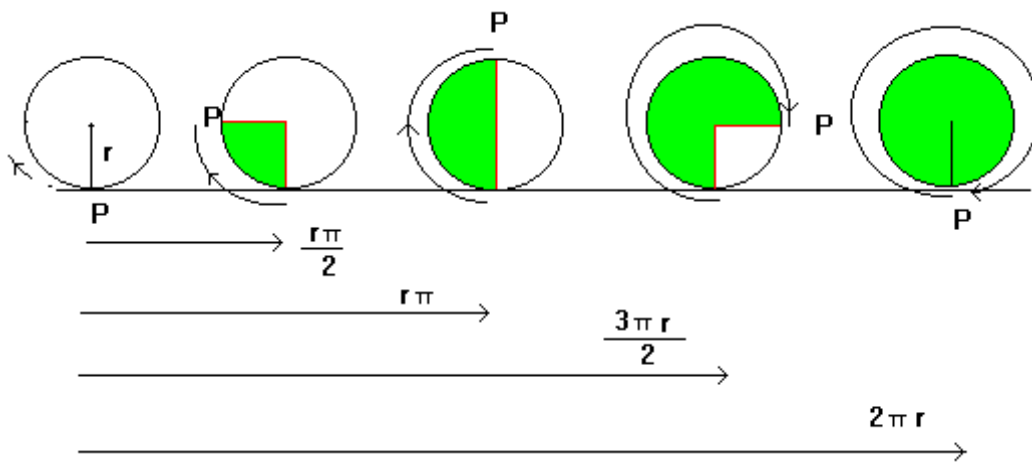
## The Equations

To develop the equations for the curve traced out by a point on the rim of the wheel we will use the angle of rotation instead of  $t$  as the parameter. We will obtain equations of the form:

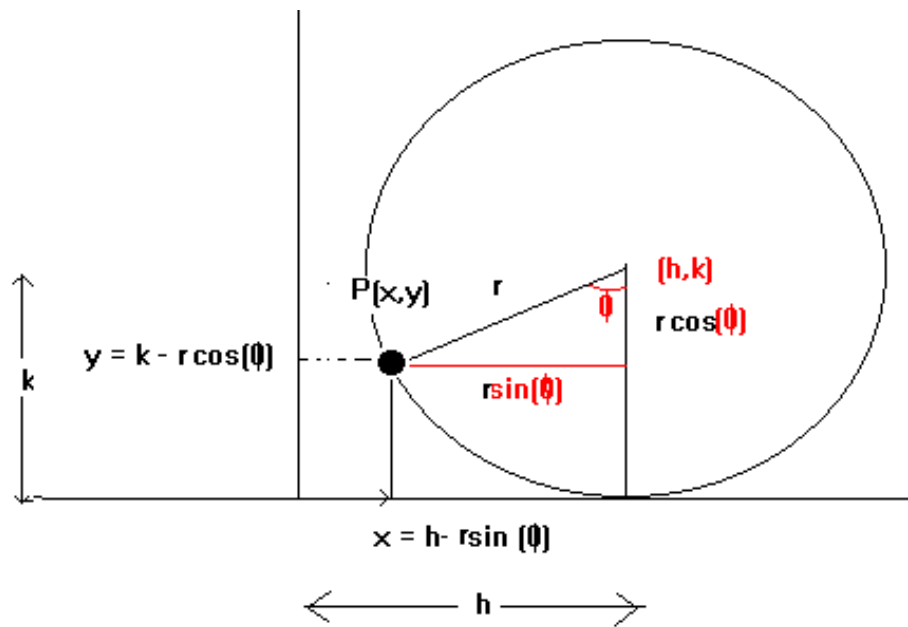
$$x = x(\phi)$$

$$y = y(\phi)$$

Suppose we start with our point  $P$  initially at the bottom of the wheel. As the wheel rotates through an angle  $\phi$  the center moves to the right a distance  $r$  (remember  $s = r \phi$ ).



Consider the coordinates of a point P after the wheel has rotated through an angle  $\phi$ . ( $\phi$  is measured clockwise).



From the diagram we see that  $x = h - r \sin(\phi)$   
 $y = k - r \cos(\phi)$

Further  $k$  is just the radius of the wheel  $k = r$  and as mentioned above  $h = r \phi$ . We obtain

$$\mathbf{x = r \phi - r \sin(\phi)}$$
$$\mathbf{y = r - r \cos(\phi)}$$