

## Graphing vector valued functions on surfaces

Again we will be using the parabaloid  $z = 1 - x^2 - y^2$  with domain the unit circle  $r = 1$  in the first quadrant

Suppose in the domain we are moving from the origin on the line parameterized by:

$$\mathbf{x}(t) := \mathbf{t} \quad \mathbf{y}(t) := \mathbf{t}$$

Then on the surface we induce the parameterization:

$$\mathbf{x}(t) := \mathbf{t} \quad \mathbf{y}(t) := \mathbf{t} \quad \mathbf{z}(t) := 1 - 2 \cdot \mathbf{t}^2$$

The only difference between graphing vector valued functions on surfaces and parametric

equations on surfaces is that we plot the position vector instead of the path in the plane.

However I'll present this as if you are not familiar with graphing parametric equations on a surface.

We start by defining the surface and putting in the Axes

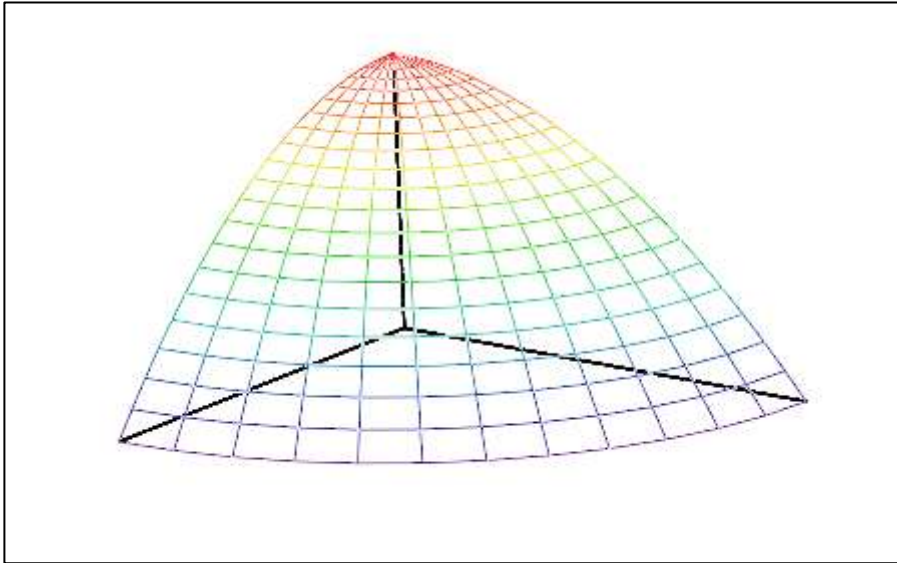
$$\mathbf{i} := 0..12 \quad \theta_{\mathbf{i}} := \pi \cdot \frac{\mathbf{i}}{24} \quad \mathbf{j} := 0..20 \quad r_{\mathbf{j}} := \frac{\mathbf{j}}{20}$$

$$\mathbf{X}_{\mathbf{i},\mathbf{j}} := r_{\mathbf{j}} \cdot \cos(\theta_{\mathbf{i}}) \quad \mathbf{Y}_{\mathbf{i},\mathbf{j}} := r_{\mathbf{j}} \cdot \sin(\theta_{\mathbf{i}}) \quad \mathbf{Z}_{\mathbf{i},\mathbf{j}} := 1 - (\mathbf{X}_{\mathbf{i},\mathbf{j}})^2 - (\mathbf{Y}_{\mathbf{i},\mathbf{j}})^2$$

Axes:

$$\mathbf{m} := 0..10 \quad \mathbf{X1}_{\mathbf{i},\mathbf{m}} := 0 \quad \mathbf{Y1}_{\mathbf{i},\mathbf{m}} := 0 \quad \mathbf{Z1}_{\mathbf{i},\mathbf{m}} := \mathbf{m} \cdot .1$$

for Plot 1 change weight to .5 to lighten the surface.



$(X, Y, Z), (X1, Y1, Z1), (Y1, Z1, X1), (Z1, X1, Y1)$

Now we define the curve and the position vector

$s := 0..FRAM1$      $t(s) := s \cdot .02$      $\tau := 0..1$      $s1 := FRAM1$

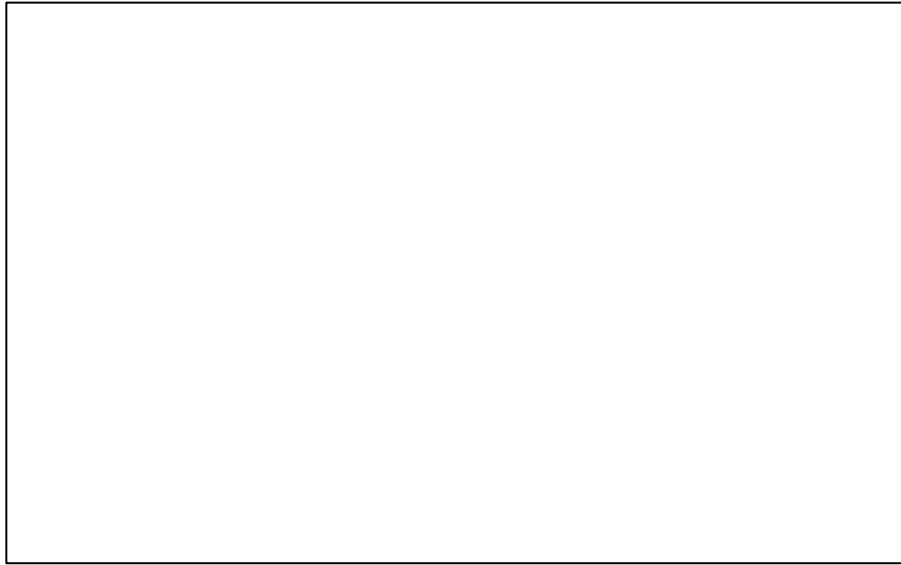
Curve on the surface     $x_{\tau, s} := t(s)$      $y_{\tau, s} := t(s)$      $z_{\tau, s} := 1 - 2 \cdot t(s)^2$

Position vector     $x1_{\tau, s1} := t(s1)$      $y1_{\tau, s1} := t(s1)$      $z1_{\tau, s1} := 1 - 2 \cdot t(s1)^2$

For Plots 5 and 6 under the appearance tab change weight to 2 to darken the curves

To animate use 35 frames since  $z=0$  when  $1 - 2 \cdot t^2 = 0$  when  $t = \frac{1}{\sqrt{2}} = .7$

$t(s) = .02s = .7$  for  $s = 35$



$(X, Y, Z), (X1, Y1, Z1), (Y1, Z1, X1), (Z1, X1, Y1), (x, y, z), (x1, y1, z1)$

Suppose in the domain we instead are following the curve  $y = t^2$

$$x(t) := t \quad y(t) := t^2$$

Then on the surface we induce the parameterization:

$$\underline{x}(t) := t \quad \underline{y}(t) := t^2 \quad z(t) := 1 - t^2 - t^4$$

$$i := 0..12 \quad \theta_i := \pi \cdot \frac{i}{24} \quad j := 0..20 \quad r_j := \frac{j}{20}$$

$$X_{i,j} := r_j \cdot \cos(\theta_i) \quad Y_{i,j} := r_j \cdot \sin(\theta_i) \quad Z_{i,j} := 1 - (X_{i,j})^2 - (Y_{i,j})^2$$

$$m := 0..10 \quad X1_{i,m} := 0 \quad Y1_{i,m} := 0 \quad Z1_{i,m} := m \cdot 1$$

Now we define the curve and the position vector

$$s := 0..FRAM1 \quad t(s) := s \cdot .02 \quad \tau := 0..1 \quad s1 := FRAM1$$

$$\text{Curve on the surface} \quad x_{\tau,s} := t(s) \quad y_{\tau,s} := t(s)^2 \quad z_{\tau,s} := 1 - t(s)^2 - t(s)^4$$

$$\text{Position vector} \quad x1_{\tau,s1} := t(s1) \quad y1_{\tau,s1} := t(s1)^2$$

$$z1_{\tau,s1} := 1 - t(s1)^2 - t(s1)^4$$

For Plots 5 and 6 under the appearance tab change weight to 2 to darken the curves

To animate use 35 frames since  $z = 0$  when  $1 - 2 \cdot t^2 = 0$  when  $t = \frac{1}{\sqrt{2}} = .7$

$t(s) = .02s = .7$  for  $s = 35$



$(X, Y, Z), (X1, Y1, Z1), (Y1, Z1, X1), (Z1, X1, Y1), (x, y, z), (x1, y1, z1)$