The Double Integral-- Cross- Sectional Method

As an example we'll set up the animation to demonstrate the volume of $f(x,y) := \cos(x) \cdot \cos(y)$ on the unit square.

We start by graphing the surface:

$a := 0 \quad b := 1 \quad c := 0 \quad d := 1 \quad \Delta x := .04 \quad \Delta y := .04 \quad i := 0 .. \frac{b-a}{\Delta x} \quad j := 0 .. \frac{d-c}{\Delta y}$

$x_i := a + i \cdot \Delta x \quad y_j := c + j \cdot \Delta y$

$f(x,y) := \cos(x) \cdot \cos(y) \quad M_{i,j} := f(x_i, y_j)$ Under Plot 1 and the Appearance Tab change to a color map with weight 3.

Now we want to create the cross- sections. In this first example we'll consider the situation where we fix y, i.e. $\int_0^1 \int_0^1 f(x,y)\,dx\,dy$.

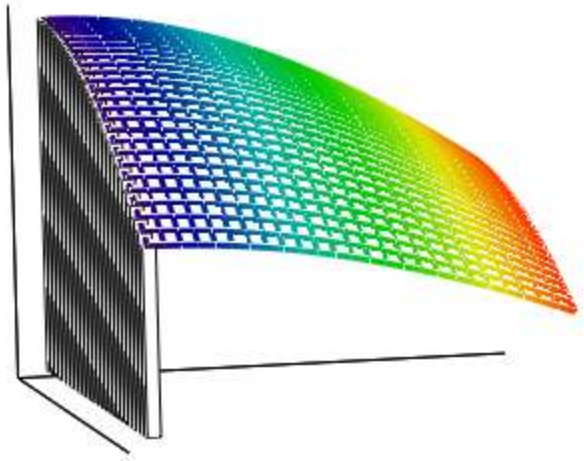
Now we define the cross sections

$s := 0 .. \frac{b-a}{\Delta x} \quad t := 0 .. \frac{d-c}{\Delta y} \cdot \frac{FRAME}{25} \quad X_t := a + t \cdot \Delta x \quad Y_s := c + s \cdot \Delta y \quad N_{s,t} := f(X_t, Y_s)$

Under the General Tab change Plot 2 from a surface plot to a bar plot. Under the Appearance tab use a solid color.

To Animate use 25 frames since Δ y = .04 and we want y to go from 0 to 1.

Set the x and y axes to go from -1 to 26 and the z axis to go from 0 to 1.



$\mathbf{M}, \mathbf{N}$

To demonstrate $\int_0^1 \int_0^1 \mathbf{f}(\mathbf{x}, \mathbf{y})\, \mathbf{dy}\, \mathbf{dx}$ simply interchange the roles of s and t.

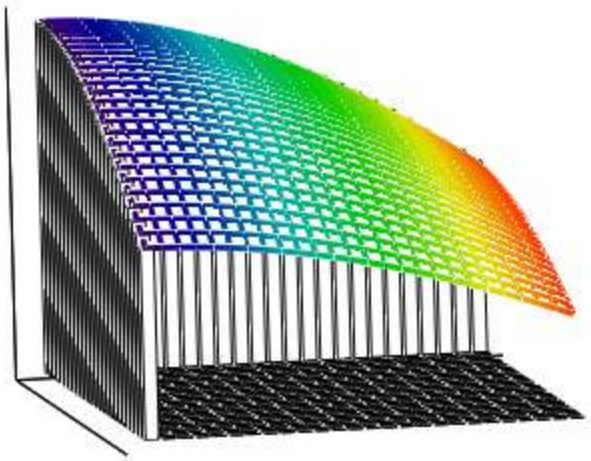
Use

$$\mathbf{s} := \mathbf{0} .. \frac{\mathbf{b} - \mathbf{a}}{\Delta \mathbf{x}} \cdot \frac{\mathbf{FRAME}}{\mathbf{25}} \qquad \mathbf{t} := \mathbf{0} .. \frac{\mathbf{d} - \mathbf{c}}{\Delta \mathbf{y}} \qquad \mathbf{X}_\mathbf{t} := \mathbf{a} + \mathbf{t} \cdot \Delta \mathbf{x} \quad \mathbf{Y}_\mathbf{s} := \mathbf{c} + \mathbf{s} \cdot \Delta \mathbf{y} \quad \mathbf{N}_{\mathbf{s},\mathbf{t}} := \mathbf{f}\left(\mathbf{X}_\mathbf{t}, \mathbf{Y}_\mathbf{s}\right)$$

Under the General Tab change Plot 2 from a surface plot to a bar plot. Under the Appearance tab use a solid color.

To Animate use 25 frames since now Δ x = .04 and we want y to go from 0 to 1.

Set the x and y axes to go from -1 to 26 and the z axis to go from 0 to 1.



M, **N**