Level Curves

Suppose T(x,y) = $1 - x^2 - y^2$ gives the temperature at any pt (x,y) in a region of the x-y plane.

Then a level curve or contour of T is a curve in the xy plane such that at every pt on that curve T has the same temperature.

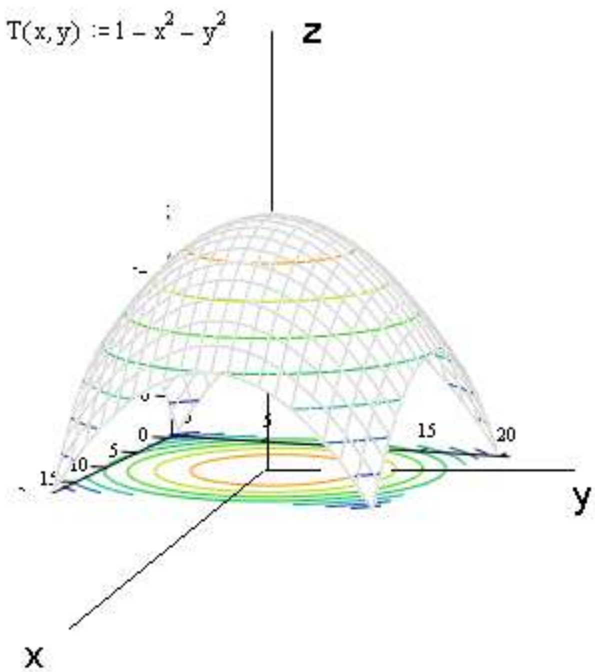
This is to say a level curve is the intersection of the surface z = f(x,y) and the horizontal plane z = k projected into the xy plane.

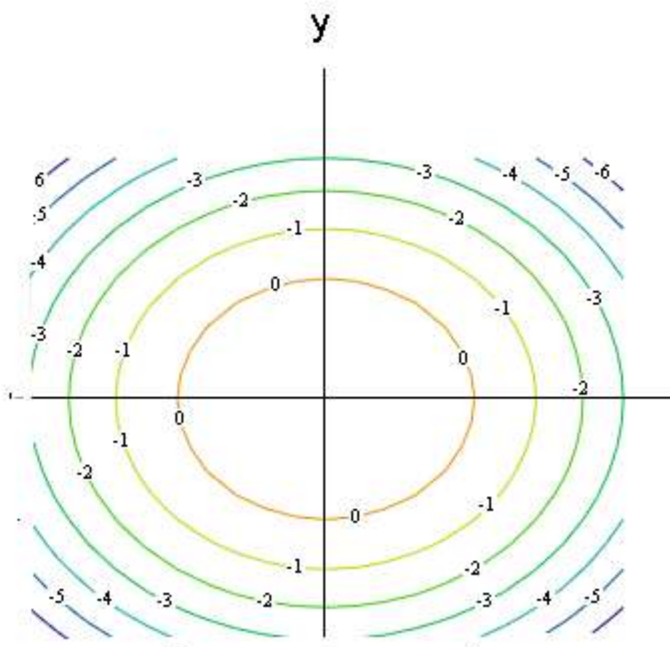
A contour diagram is a set of level curves for values of z in equal increments.

Note for the function T(x,y) = $1 - x^2 - y^2$ the contours z =k are the circles $x^2 + y^2 = 1 - k$

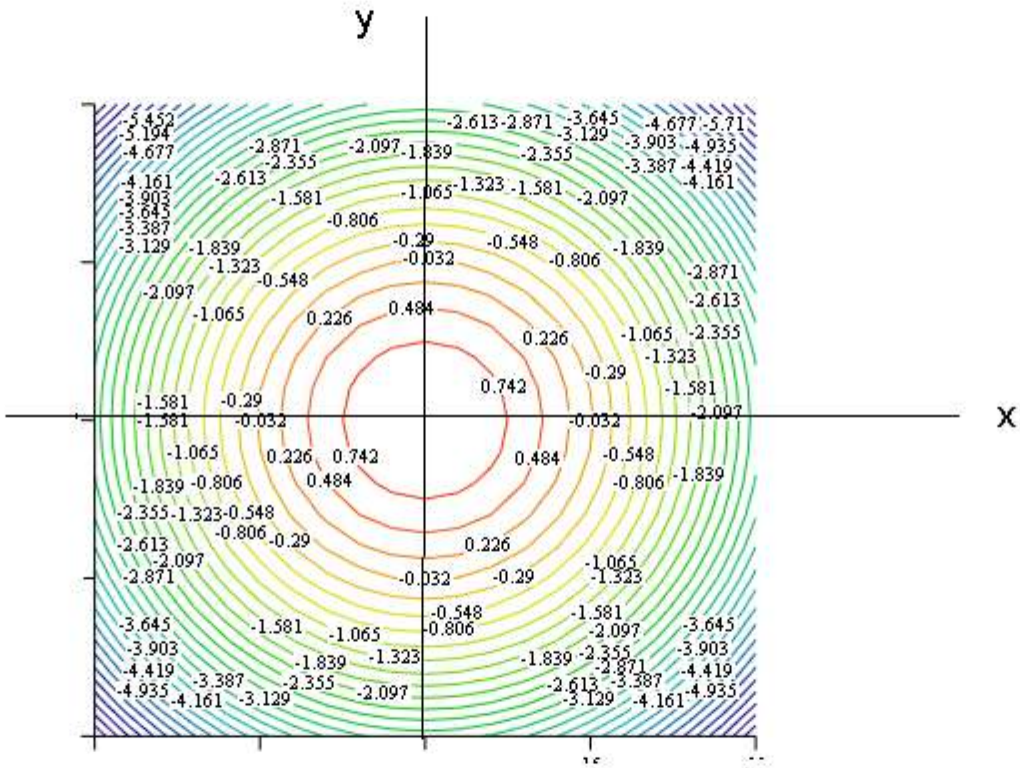
If k = 1 we have the single point (0,0). As k decreases we obtain concentric circles.

See the diagrams below:

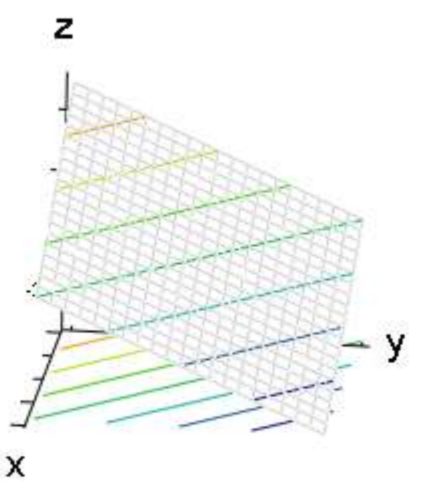


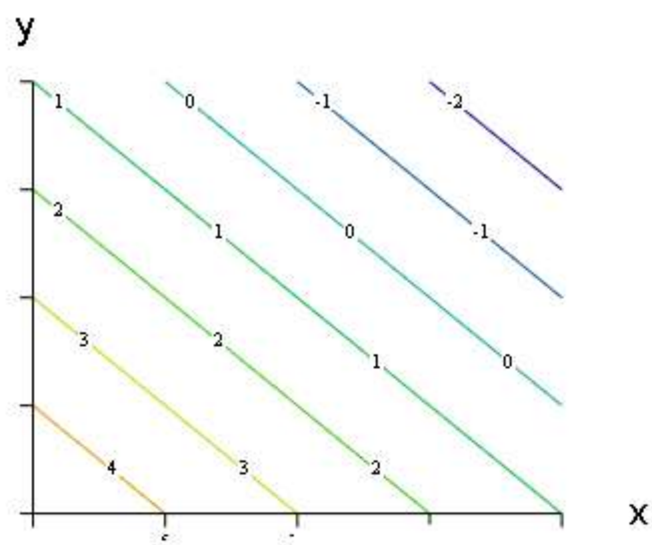


Observation 1 The closer the contours are the steeper the surface.

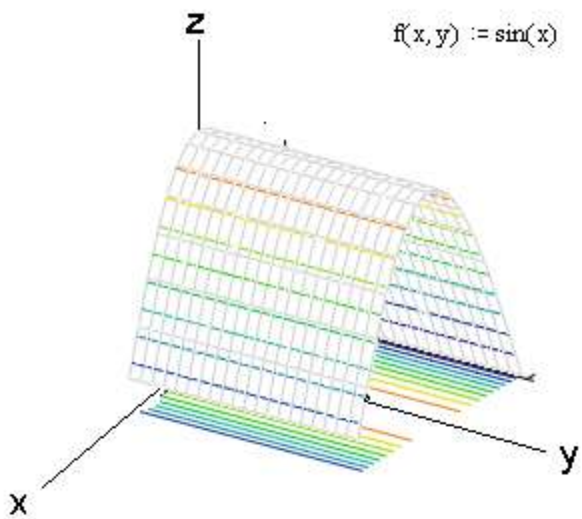


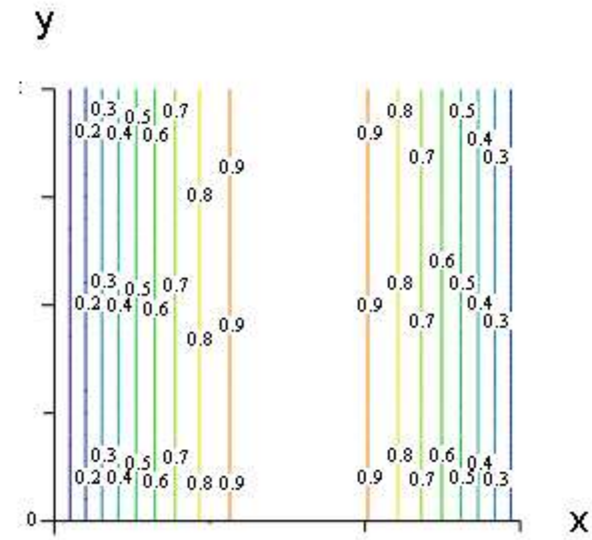
Let's consider the contour diagram of a plane





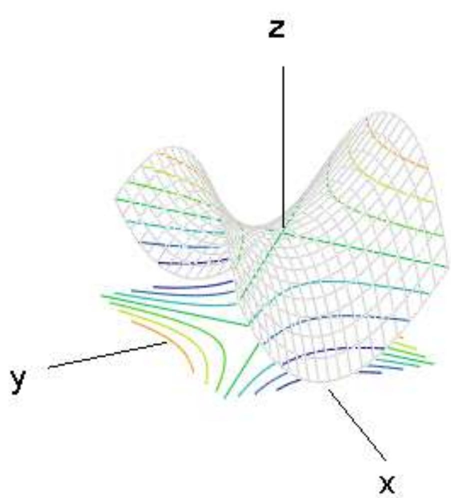
<u>Observation 2</u> The contours are equally spaced lines for planes. What we are saying is that the amount z changes in a particular direction are the same between any two contours.
Be Careful just because the contour diagram consists of lines it doesn't necessarily mean the surface is a plane. The contours must also be equally spaced as the following diagram shows.

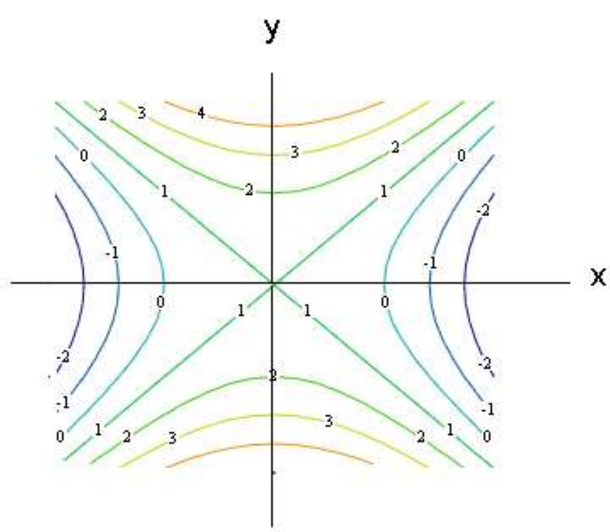




Let's consider a saddle:

$f(x, y) := 1 - x^2 + y^2$





<u>Observation 3</u> Starting at the center for fixed x, z increases as y increases or decreases.
For fixed y, z decreases as x increases or decreases.

To see this we start with

$$k = 1 - x^2 + y^2$$

$$x^2 - y^2 = 1 - k$$

If k < 1 then we have a hyperbola opening to the right and left.

If k = 1 then we have $x^2 - y^2 = 0$ which give the lines y = $\pm$ x.

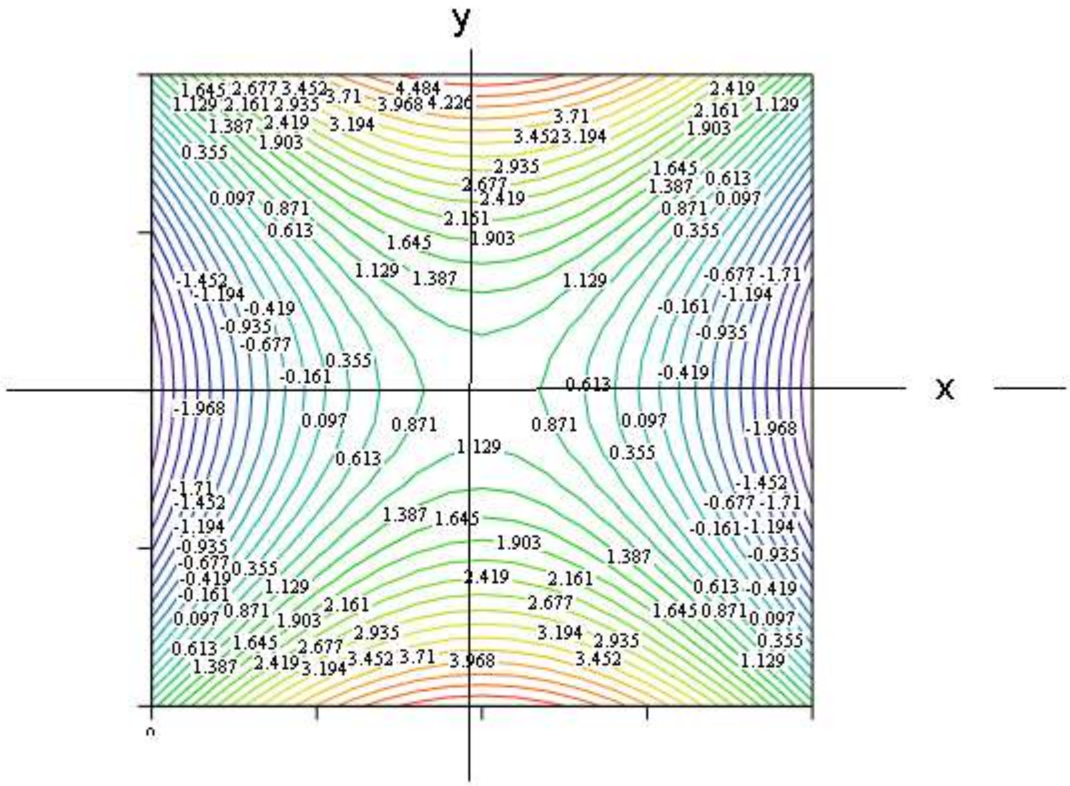
If k > 1 then we have $y^2 - x^2 = k - 1$ which are hyperbolas opening up and down.

Consider the matrix of values for this surface below and compare it to the corresponding contour diagram

y

x

	6	7	8	9	10	11	12	13	14	15
2	-0.92	-1.2	-1.4	-1.52	-1.56	-1.52	-1.4	-1.2	-0.92	-0.56
3	-0.32	-0.6	-0.8	-0.92	-0.96	-0.92	-0.8	-0.6	-0.32	0.04
4	0.2	-0.08	-0.28	-0.4	-0.44	-0.4	-0.28	-0.08	0.2	0.56
5	0.64	0.36	0.16	0.04	0	0.04	0.16	0.36	0.64	1
6	1	0.72	0.52	0.4	0.36	0.4	0.52	0.72	1	1.36
7	1.28	1	0.8	0.68	0.64	0.68	0.8	1	1.28	1.64
8	1.48	1.2	1	0.88	0.84	0.88	1	1.2	1.48	1.84
9	1.6	1.32	1.12	1	0.96	1	1.12	1.32	1.6	1.96
10	1.64	1.36	1.16	1.04	1	1.04	1.16	1.36	1.64	2
11	1.6	1.32	1.12	1	0.96	1	1.12	1.32	1.6	1.96
12	1.48	1.2	1	0.88	0.84	0.88	1	1.2	1.48	1.84
13	1.28	1	0.8	0.68	0.64	0.68	0.8	1	1.28	1.64
14	1	0.72	0.52	0.4	0.36	0.4	0.52	0.72	1	1.36
15	0.64	0.36	0.16	0.04	0	0.04	0.16	0.36	0.64	1
16	0.2	-0.08	-0.28	-0.4	-0.44	-0.4	-0.28	-0.08	0.2	0.56
17	-0.32	-0.6	-0.8	-0.92	-0.96	-0.92	-0.8	-0.6	-0.32	0.04



Using Contour Diagrams to graph implicitly defined functions

Suppose we have the implicitly defined function cos(xy) = y. How do we generate its graph?

We rewrite it as cos(xy) - y = 0 and can then think of the graph as a level curve of
f(x,y) = cos(xy) - y corresponding to the value 0.

In the graph below Note we also have the graphs of cos(xy) = y + c for c =-4,4,-2, and 2 also.

$f(x, y) := \cos(x \cdot y) - y$