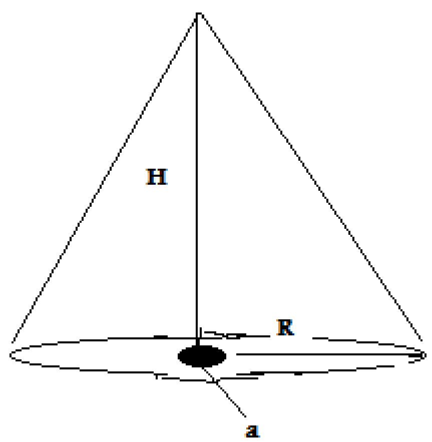
Suppose we have an inverted conical tank with height H and radius R

Suppose fluid is flowing through a hole in the bottom with cross sectional area a

with velocity given by $V(t) = k [2 g h(t)]^{1/2}$ where h(t) is the height of fluid in the tank.

Find the time required to empty the tank



Since $V(t) = k\cdot\sqrt{2g\cdot h(t)}$ the flux through the circle of crossection a is

$\Phi = k\cdot\sqrt{2g\cdot h}\cdot a$ Recall the flux is just the volume of fluid per unit time flowing through the cross-section at the bottom

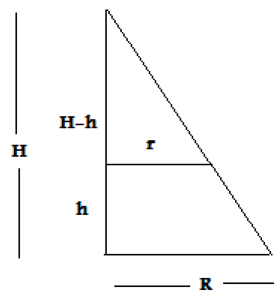
Let v be the volume then:

$$\frac{dv}{dt} = k\cdot\sqrt{2g\cdot h}\cdot a \qquad (1)$$

(It is important to note for later that k < 0 as the velocity must be downward).

On the other hand :

$$v = \frac{1}{3}\cdot\pi\cdot r^2\cdot h$$



Using similar triangles $r = (H - h)\cdot\frac{R}{H}$

We obtain $v = \frac{1}{3}\cdot\pi\cdot\left(\frac{R}{H}\right)^2\cdot(H-h)^2\cdot h = \beta\cdot\left[(H-h)^2\cdot h\right]$

Where $\beta = \frac{1}{3}\cdot\pi\cdot\left(\frac{R}{H}\right)^2$

$$\frac{dv}{dt} = \beta\cdot(H - 3\cdot h)\cdot(H - h)\cdot\frac{dh}{dt} \qquad (2)$$

Set (1) = (2) We obtain the IVP

$$\beta\cdot(H - 3\cdot h)\cdot(H - h)\cdot\frac{dh}{dt} = k\cdot\sqrt{2g\cdot h}\cdot a \qquad \text{with } h(0) = H$$

Separating

$$\frac{(H - 3\cdot h)\cdot(H - h)}{\sqrt{h}}\cdot dh = \frac{k\cdot\sqrt{2g}\cdot a}{\beta}\cdot dt = \alpha\cdot dt$$

Where $\alpha = \frac{k\cdot\sqrt{2g}\cdot a}{\beta}$

$$h^{\frac{5}{2}}\cdot\left(\frac{2\cdot H^2}{h^2}-\frac{8\cdot H}{3\cdot h}+\frac{6}{5}\right)=\alpha t+C$$

$$h^{\frac{5}{2}}\cdot\left(\frac{2\cdot H^2}{h^2}-\frac{8\cdot H}{3\cdot h}+\frac{6}{5}\right)=\alpha t+\frac{8\cdot H^{\frac{5}{2}}}{15}$$

Now to get the time required to empty the tank set $h = 0$ and remember k and therefore

α are negative

$$t=\frac{-8\cdot H^{\frac{5}{2}}}{15\cdot\alpha}=-\frac{4\cdot\sqrt{2}\cdot H^{\frac{5}{2}}\cdot\beta}{15\cdot a\cdot\sqrt{g}\cdot k}=-\frac{4\cdot\pi\cdot\sqrt{2}\cdot\sqrt{H}\cdot R^2}{45\cdot a\cdot\sqrt{g}\cdot k}$$

For example

$H = 5$ $\quad a = 1$ $\quad R = 2$ $\quad k = -0.2$

$t = 5.6$