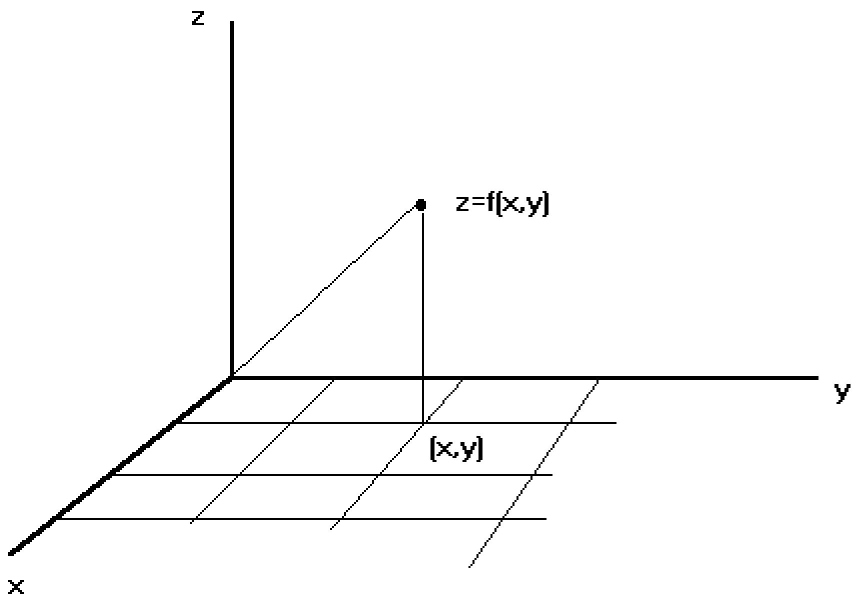
Lab 1 Introduction to Graphing Functions of 2 Variables

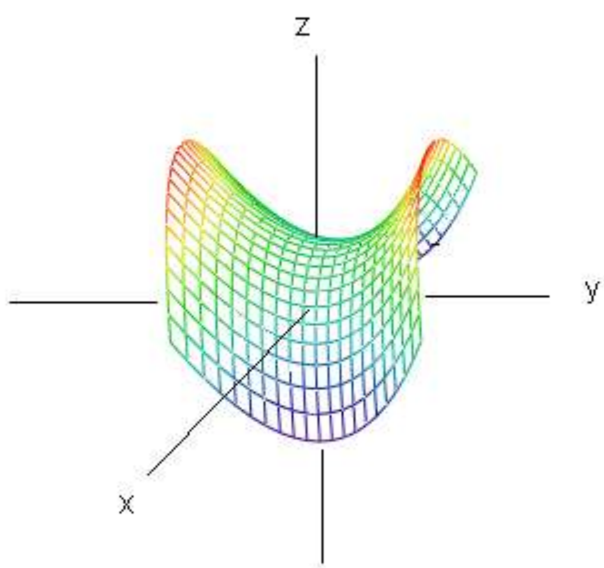
A function of 2 variables is a function of the form z = f(x,y) such that for each point (x,y) in some region of the plane there is one and only one value of z.

See figure 1 below.

Figure 1



The graph of a function is the set of all points (x,y,f(x,y)) which generates a 2 dimensional surface in 3-space. This is also called a lamina.



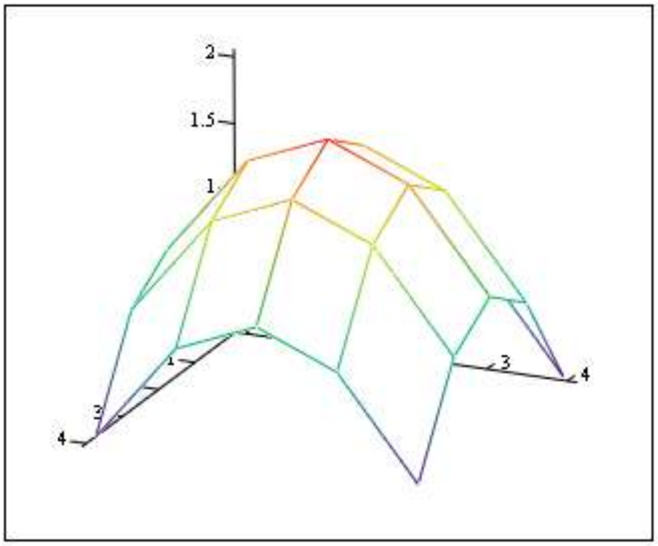
In Mathcad (for now) we can only graph over a rectangular domain. Further when Mathcad plots the graph it evaluates f(x,y) at the points you tell it to, then connects successive points with straight line segments.

For example suppose we plot the parabaloid $f(x,y) = 1- x^2- y^2$ on the rectangle [0,1] x [0,1] with a mesh size of .2 in the x direction and .2 in the y direction. (Note Mathcad does not include the coordinate axes.) For now don't worry about the formatting we'll explain it shortly'

$a := -1 \quad b := 1 \quad c := -1 \quad d := 1 \quad \Delta x := .5 \quad \Delta y := .5$

$i := 0 .. \frac{b-a}{\Delta x} \quad j := 0 .. \frac{d-c}{\Delta y} \quad x_i := a + i \cdot \Delta x \quad y_j := c + j \cdot \Delta y$

$f(x, y) := 2 - x^2 - y^2 \quad M_{i,j} := f(x_i, y_j)$



M

M defines the matrix of z values. To see theses values simply type M and hit =

$$M = \begin{bmatrix} 0 & 0.75 & 1 & 0.75 & 0 \\ 0.75 & 1.5 & 1.75 & 1.5 & 0.75 \\ 1 & 1.75 & 2 & 1.75 & 1 \\ 0.75 & 1.5 & 1.75 & 1.5 & 0.75 \\ 0 & 0.75 & 1 & 0.75 & 0 \end{bmatrix}$$

Note Mathcad starts counting at 0 so the columns are numbered 0 to 4 not 1 to 5 ; the same for the rows so x_3 is the fourth row and y_1 is the second column so $M_{3,1}$ is the matrix element 4th row 2d column

Since we traditionally plot a 3 D graph with x coming out of the page the columns correspond to the x values for fixed y and the rows correspond to the y values for fixed x. For example

$x_3 = 0.5 \quad y_1 = -0.5 \quad M_{3,1} = 1.5$

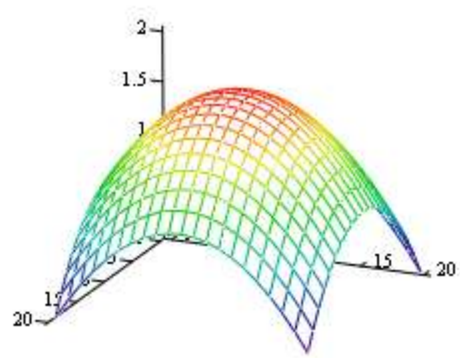
since $f(x, y) := 2 - x^2 - y^2 \quad f(.5, -.5) = 1.5$

By decreasing the step size to say 0.1 we smooth out the surface as is seen below

$a := -1 \quad b := 1 \quad c := -1 \quad d := 1 \quad \Delta x := .5 \quad \Delta y := .5$

$i := 0 .. \frac{b - a}{\Delta x} \quad j := 0 .. \frac{d - c}{\Delta y} \quad x_i := a + i \cdot \Delta x \quad y_j := c + j \cdot \Delta y$

$f(x, y) := 2 - x^2 - y^2 \quad M_{i,j} := f(x_i, y_j)$



M

The Template you'll be using for all graphs is :

1. First we need to define the interval in the x direction, [a,b] the interval in the y direction [c,d] and the step sizes Δx and Δy We do this simply by:

$a := \blacksquare \quad b := \blacksquare \quad c := \blacksquare \quad d := \blacksquare \quad \Delta x := \blacksquare \quad \Delta y := \blacksquare$ Simply fill these in for the particular example.

Use the := in all definitions not =

2. We need to create the points at which to evaluate f(x,y)

$i := 0 .. \frac{b - a}{\Delta x} \quad j := 0 .. \frac{d - c}{\Delta y} \quad x_i := a + i \cdot \Delta x \quad y_j := c + j \cdot \Delta y$ Here do nothing as all these quantities are calculated automatically from the data furnished in 1. above.

3. Define f(x,y) and the Matrix M

$f(x, y) := \blacksquare \qquad M_{i,j} := f(x_i, y_j)$ only fill in the function f(x,y) , M will be computed automatically

Once you've typed in the template all you have to do is copy, paste and make appropriate changes for all subsequent graphs so really you only have to do the work one time . Caution: use different letters for M as you go from one example to another otherwise if for your second example the array is smaller you'll end up with a hybrid of the 2 examples like something from " The Island of Dr Moreau" .

One more example and then you're on your own. Let's graph $f(x, y) := |\sin(x)| \cdot |\cos(y)|$ on the rectangle

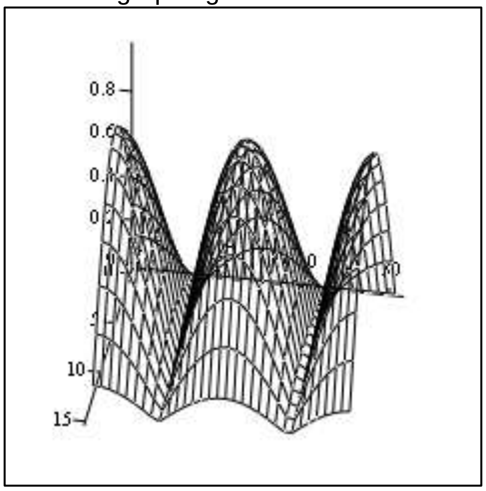
[0,π]x[-π,π] with a step size of .2 in both the x and y direction.

$a := 0 \quad b := \pi \quad c := -\pi \quad d := \pi \quad \Delta x := .2 \quad \Delta y := .2$

$i := 0 .. \frac{b-a}{\Delta x} \quad j := 0 .. \frac{d-c}{\Delta y} \quad x_i := a + i \cdot \Delta x \quad y_j := c + j \cdot \Delta y$

$f(x, y) := |\sin(x)| \cdot |\cos(y)| \quad N_{i,j} := f(x_i, y_j)$

From the graphing Menu choose the surface graph and put N in the place holder on bottom



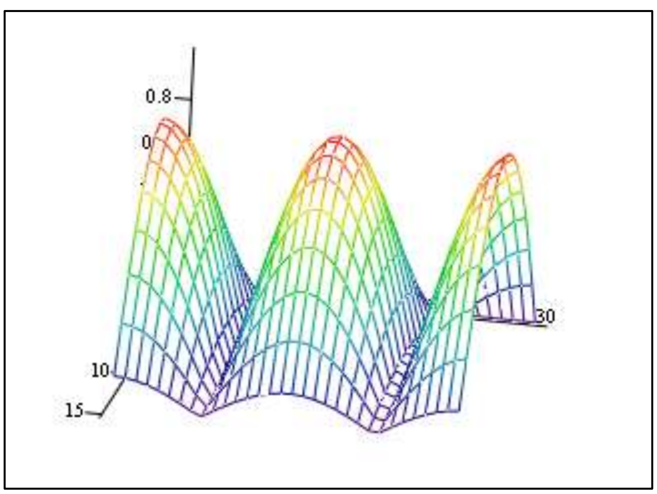
N

Let's format this graph

1. double click on the graph and your FORMAT window will appear

2. We make the following changes under APPEARANCE choose COLOR MAP and HIDE LINES.

You'll notice that in the first graph you're seeing all the points-HIDE LINES is like putting a thin veneer on the graph



N

You can view the graph from different perspectives by left clicking on the graph and moving the mouse.
Notice there are several options and windows --the best way to learn is to experiment with the various features in the FORMAT window.

Exercise 1
Graph f(x,y) = $10-x^2-y^2$ on the rectangle [-10,10] x [-5,5] with a step size of .1 in both the x and y directions.

Exercise 2
Graph

$$f(x,y) := \frac{1}{1+\sqrt{|y-\sin(x)|}+\sqrt{|x+\sin(y)|}}$$

over the square [-3,3] x[-3,3] with a step size of $\Delta x = \Delta y =$ **.1**

Exercise 3 A Cylindrical Surface
Graph f(x,y) = cos(y)
on the rectangle [-1,1] x[0,4π] with a step size of **.1** in the x direction and **.2** in the y direction.

Exercise 4
Graph f(x,y) = |x||y| on the square [-1,1] x[-1,1] with a step-size of **.1** in both directions. Consider the graph for various rotations.

Graphing Multiple Functions

We use exactly the same template as before we simply add one more function and one more array.

For example suppose we want to graph the 2 parabaloids $f(x,y) := 1 - x^2 - y^2$ and $g(x,y) := x^2 + y^2$

on the same graph on the square [-1,1]x[-1,1] with a step size of .1.

I'll name the 3 arrays simply M1 and M2

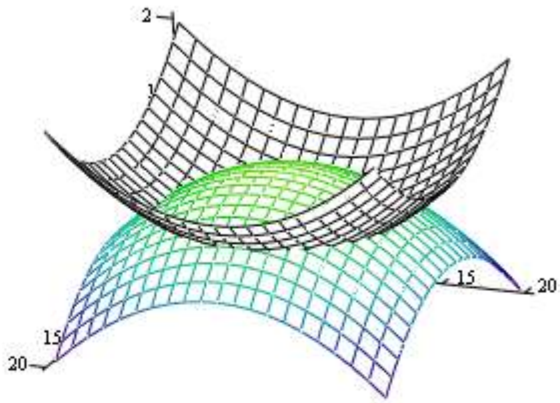
$$a := -1 \quad b := 1 \quad c := -1 \quad d := 1 \quad \Delta x := .1 \quad \Delta y := .1$$

$$i := 0 .. \frac{b-a}{\Delta x} \quad j := 0 .. \frac{d-c}{\Delta y} \quad x_i := a + i \cdot \Delta x \quad y_j := c + j \cdot \Delta y$$

$$f(x,y) := 1 - x^2 - y^2 \quad M1_{i,j} := f(x_i, y_j) \quad g(x,y) := x^2 + y^2 \quad M2_{i,j} := g(x_i, y_j)$$

On the graph Once you put in M1 hit the comma and a box will appear to put in M2.

You can format separately in FORMAT window with PLOT 1 corresponding to M1 and PLOT 2 corresponding to M2



M1, M2

I have used a color plot for f and a black map for g. Also under the GENERAL tab I've clicked off SHOW BORDER.

Instead of me assigning exercises use your imagination and come up with your own examples. By the way you can graph as many fns as you want by adding more fns and more arrays like we just did.

For some of the parametric Surface Plots and curves I've used in Lecture I've had as many as 15 on one graph. See the animations in the Directional Derivative lecture for example.