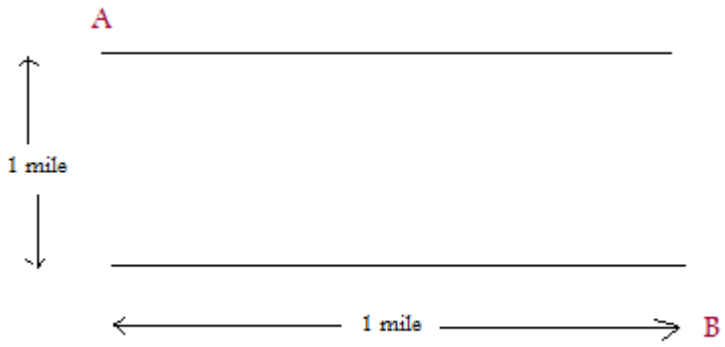
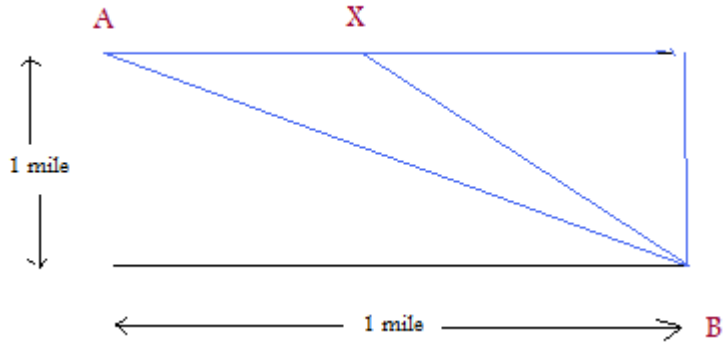
A bird flies home

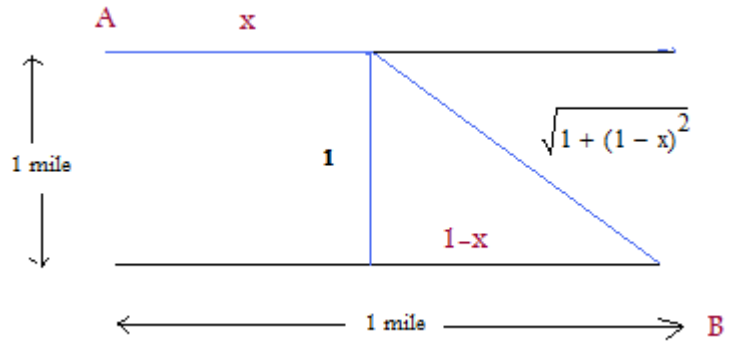
Suppose a bird can fly 35 mph over land and 15 mph over water. Suppose the bird wants to get to a point across a river 1mile wide and 1 mile down river. What path should the bird take to reach home in the least amount of time?



The bird could fly directly from A to B which would be the shortest distance but at the slowest speed. It could also fly a mile along the bank and cut across but this would be the longest possible distance. Is there a point x such that if it flew along the bank to pt x and then cut across
it could minimize the time?



We start by finding an expression for the time:



$$t(x) := \frac{x}{35} + \frac{\sqrt{1 + (1 - x)^2}}{15}$$

If it flies over just water x = 0 and $t(0) = 0.094$

If it flies 1 mile along the bank and cuts across x = 1 $t(1) = 0.095$

For all other possibilities we find dt/dx and set it equal to 0.

$$\frac{x}{35} + \frac{\sqrt{1 + (1 - x)^2}}{15}$$

by differentiation, yields

$$\frac{2 \cdot x - 2}{30 \cdot \sqrt{(x - 1)^2 + 1}} + \frac{1}{35} = 0$$

has solution(s)

$$1 - \frac{3 \cdot \sqrt{10}}{20} = 0.526$$

$t(.526) = 0.089$

I the bird flies .526 miles then flies to point B it minimizes the time. See Animation Bird

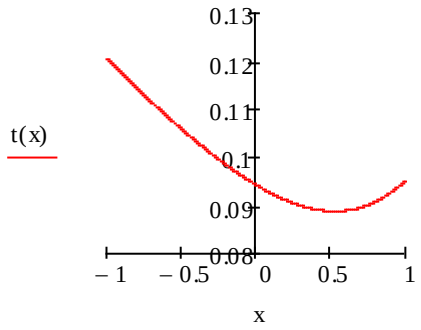
Using the Mathcad minimize function

Create a solve block by simply typing the word Given in a math region

Given

$$t(x) = \frac{x}{35} + \frac{\sqrt{1 + (1 - x)^2}}{15}$$

Make an initial guess you can use a graph to find such a guess



x := .5

Use the built in minimize function

M := Minimize(t, x) M = 0.526 t(M) = 0.089