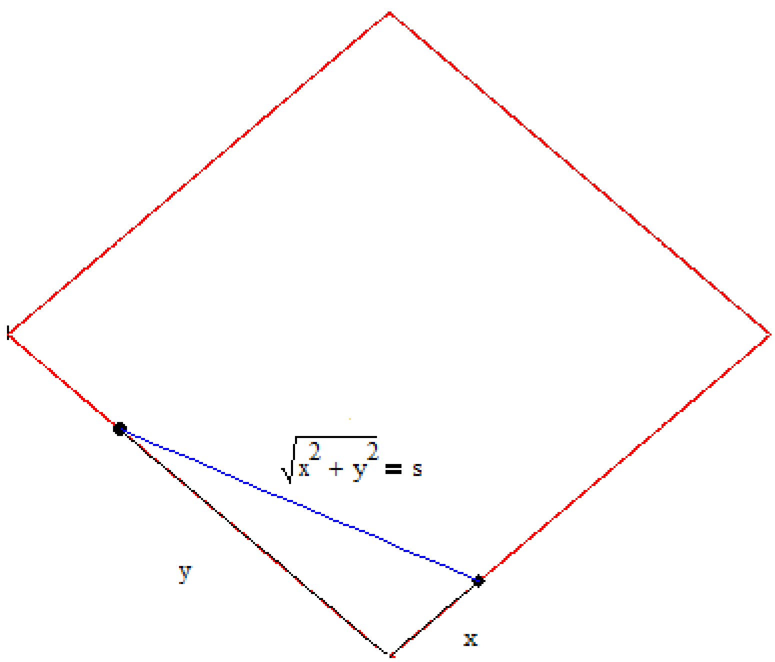
The following related rates problems deal with baseball.

A baseball diamond is 90 ft on a side.

Example 1 A ball is hit toward third base at 90 ft/sec. The batter runs toward first base at 30ft/sec.

At what rate is the distance between the ball and runner changing when the runner is 30 ft down the line?

See the animation 3rd base



From the diagram the distance s from the runner to the ball is:

$$s^2 = x^2 + y^2$$

$$2s \cdot \frac{ds}{dt} = 2 \cdot x \cdot \frac{dx}{dt} + 2 \cdot y \cdot \frac{dy}{dt}$$

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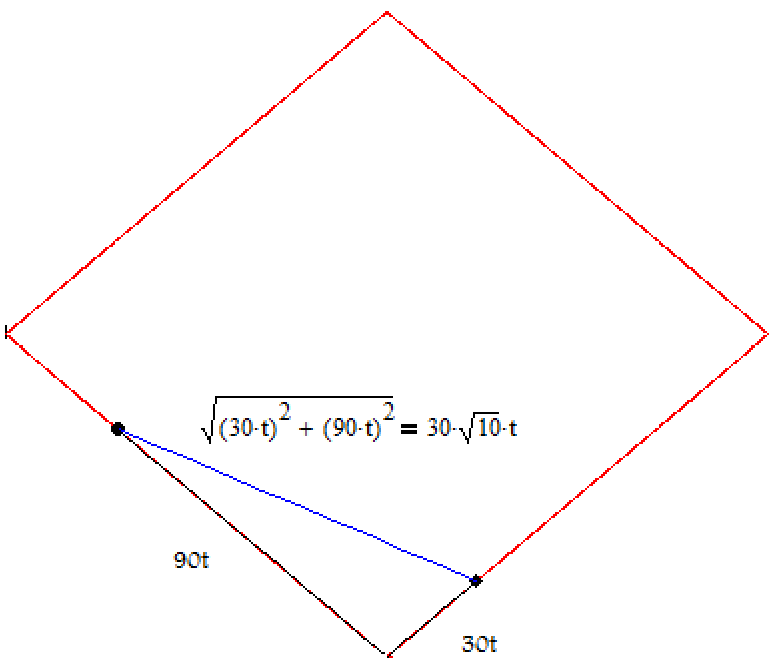
at t= 1 the runner is 30 ft down the line and the ball is 90 ft down the third base line

$$s \cdot \frac{ds}{dt} = 30 \cdot 30 + 90 \cdot 9$$

$$s = \sqrt{30^2 + 90^2}$$

$$\frac{ds}{dt} = \frac{30^2 + 90^2}{\sqrt{30^2 + 90^2}} = 94.86$$

Alternatively



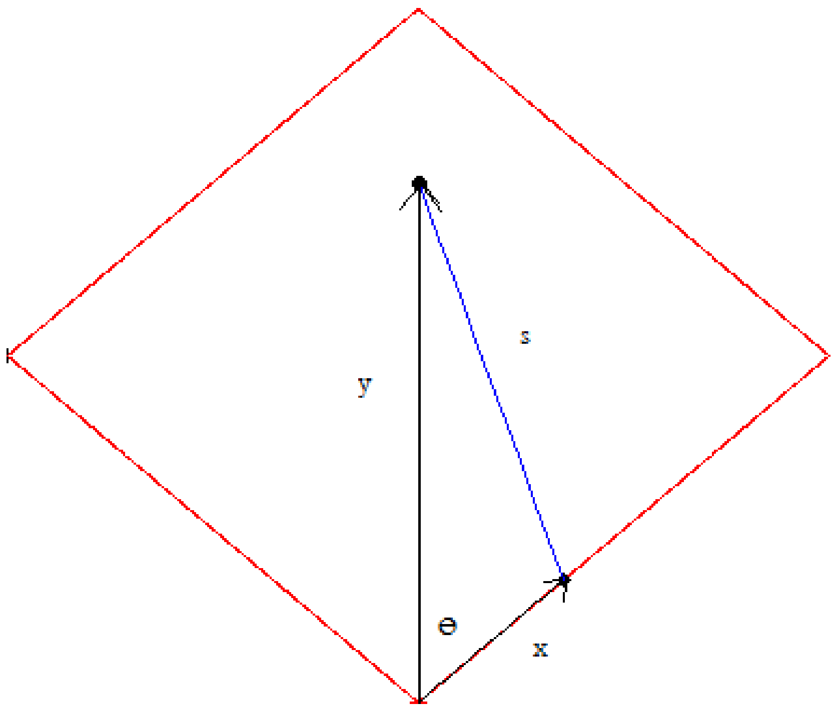
$$s = 30 \cdot \sqrt{10} \cdot t$$

$$\frac{ds}{dt} = 30 \cdot \sqrt{10} = 94.868 \frac{ft}{s}$$

Example 2 This time a ball is hit toward second base at 90 ft/sec. The batter runs toward first base at 30ft/sec.

At what rate is the distance between the ball and runner changing when the runner is 30 ft down the line?

See the animation 2d base



The difficulty here is that we no longer have a right triangle so we must use the Law of Cosines

$$s^2 = x^2 + y^2 - 2 \cdot x \cdot y \cdot \cos(\theta)$$

At t = 1 x = 30 y = 90 θ has the constant value π /4.

$$s \cdot \frac{ds}{dt} = x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt} - \frac{dx}{dt} \cdot y \cdot \frac{\sqrt{2}}{2} - x \cdot \frac{dy}{dt} \cdot \frac{\sqrt{2}}{2}$$

$$s \cdot \frac{ds}{dt} = 30^2 + 90^2 - 30 \cdot 90 \cdot \frac{\sqrt{2}}{2} - 30 \cdot 90 \cdot \frac{\sqrt{2}}{2}$$

$$s \cdot \frac{ds}{dt} = 5.182 \times 10^3$$

when x = 30 and y = 90 we can calculate s from $s = \sqrt{30^2 + 90^2 - 2 \cdot 30 \cdot 90 \cdot \cos\left(\frac{\pi}{4}\right)} = 71.98$

$$\frac{ds}{dt} = \frac{5.182 \times 10^3}{71.983} = 72 \frac{ft}{s}$$

Alternatively

$$s^2 = (30 \cdot t)^2 + (90 \cdot t)^2 - 2 \cdot 30 \cdot t \cdot 90 \cdot t \cdot \frac{\sqrt{2}}{2}$$

$$2s \cdot \frac{ds}{dt} = 18000 \cdot t - 5400 \cdot \sqrt{2} \cdot t$$

at t = 1

$$\frac{ds}{dt} = \frac{18000 - 5400 \cdot \sqrt{2}}{2 \cdot 71.983} = 72 \frac{ft}{s}$$