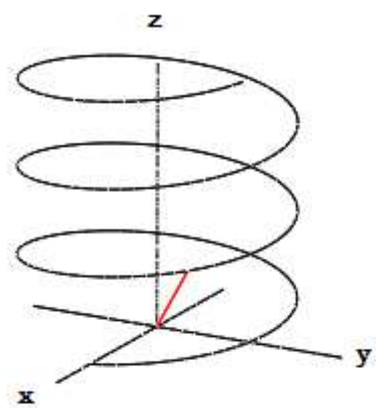
An Example of Unit Tangents and Unit Normals in 3D

Consider the helix

$$\vec{r}(t) = \cos(t)\cdot\vec{i} + \sin(t)\cdot\vec{j} + t\cdot\vec{k}$$

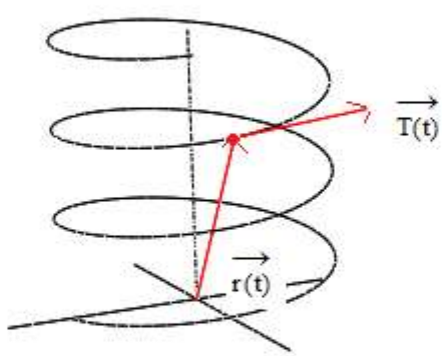


Then the velocity vector is

$$\vec{v}(t) = -\sin(t)\cdot\vec{i} + \cos(t)\cdot\vec{j} + \vec{k}$$ Note $\left|\vec{v}(t)\right| = \sqrt{2}$ so the speed is constant

The unit tangent is

$$\vec{T}(t) = \frac{1}{\sqrt{2}}\cdot\left(-\sin(t)\cdot\vec{i} + \cos(t)\cdot\vec{j} + \vec{k}\right)$$



The acceleration vector is:

$\vec{a(t)} = -\cos(t)\cdot\vec{i} + \sin(t)\cdot\vec{j}$ since the velocity is constant and $\left|\vec{a(t)}\right| = 1$ this is also the unit normal vector and a_N = 1 and a_T = 0.

Note that $\vec{N(t)}$ points directly at the z axis in a horizontal plane at all times t.

See Animation 1 This shows the position velocity and acceleration on the helix

See Animation 2 This simultaneously generates the helix while showing the position,unit tangent and unit normal vectors