Question 1

$$f(x,y) = x^2 + 2\cdot y^2 - x^2\cdot y$$

Critiacl pts occur where $\nabla f = 0$ or where f_x and f_y are simultaneously 0

$$f_x = 2\cdot x - 2\cdot x\cdot y = 2x\cdot(1 - y)$$

$$f_y = 4\cdot y - x^2$$

$f_x = 0$ if x = 0 or y = 1

using these in the equation for $f_y = 0$:

If x= 0 y = 0

If y = 1 then $4 - x^2 = 0$ so x = ±2

Therefore we have (0,0) , (2,1), and (-2,1)

Use the 2d partials test to classify

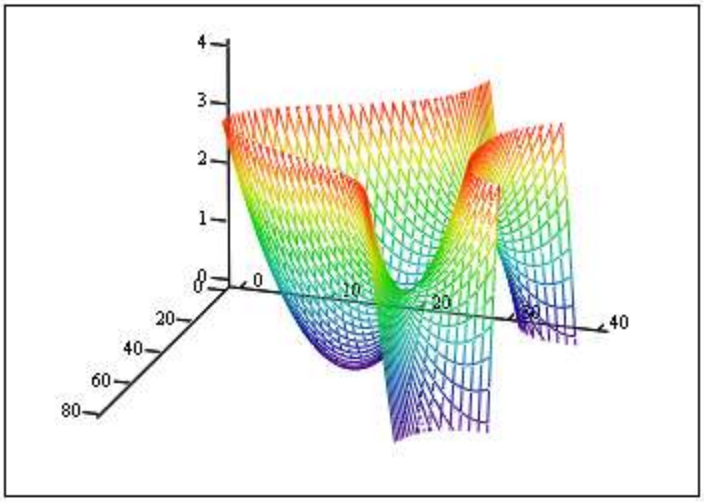
$$D = f_{xx}\cdot f_{yy} - \left(f_{xy}\right)^2 = (2 - 2y)\cdot(4) - (-2x)^2$$

At (0,0) $D = 8$ and $f_{xx} = 2$ therefore you have a minimum

At (2,1) $D = -16$ therefore you have a saddle point.

At (-2,1) $D = -16 = 0$ therefore you have a saddle point.

See the graph and contour diagram below



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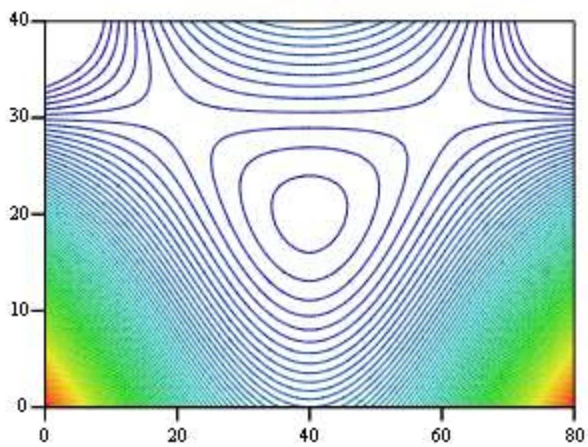

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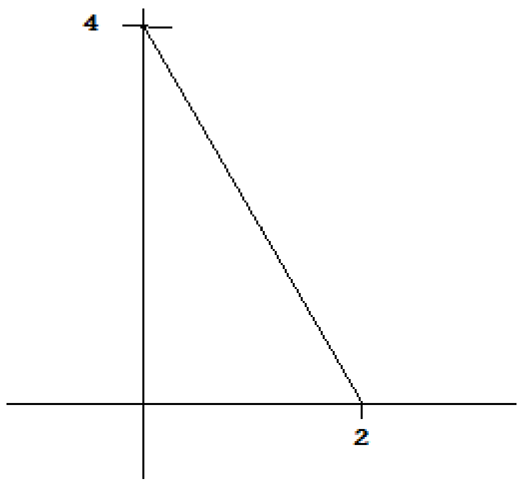
Question 2

$f(x,y) := x \cdot y - x - y + 3$

$f_x = y - 1$

$f_y = x - 1$

The only critical point is (1,1) which you can easily verify with the second partials test



From (0,0) t0 (2,0)   y = 0   f(x,y) = 3- x   the max is 3 at x = 0 and the min is 1 at x= 2

From (0,0) to (0,4)   x = 0   f(x,y) = -y+3   the max is 3 at y = 0 and the min is -1 at y = 4

From (2,0) to (4,0)   y = -2x +4 $f(x,y) = 5 \cdot x - 2 \cdot x^2 - 1$ which is a function of a single variable x let's call it g(x)

when x= 2 $g(2) = 1$ when x= 0 $g(0) = -1$ the critical point of g is x = 5/4

g(5/4) = 17/8

The Max on the boundary is 3 at (0,0) the min is -1 at (0,4)

comparing with the interioir pt (1,1) f(1,1) =2 therefore the max and min are on the boundary.